Fluid-Structure Interactions from the Large to the Very Small

Michael Shelley, New York University
October 16, 2013
Fluid-Structure Interactions from the Large to Very Small

Michael Shelley
Courant Institute, NYU

Simons Foundation Science Series
The Reynolds Number: \( Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho UL}{\mu} \)

E.M. Purcell
Life at Low Reynolds Number
<table>
<thead>
<tr>
<th>A self-propelled organism</th>
<th>Its Reynolds Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>A large whale swimming at 10 m/s</td>
<td>300,000,000</td>
</tr>
<tr>
<td>A tuna swimming at the same speed</td>
<td>30,000,000</td>
</tr>
<tr>
<td>A duck flying at 20 m/s</td>
<td>300,000</td>
</tr>
<tr>
<td>A large dragon fly going 7 m/s</td>
<td>30,000</td>
</tr>
<tr>
<td>A copepod in a speed burst of 0.2 m/s</td>
<td>300</td>
</tr>
<tr>
<td>Flapping wings of the smallest flying insects</td>
<td>30</td>
</tr>
<tr>
<td>An invertebrate larva, 0.3 mm long, at 1 mm/s</td>
<td>0.3</td>
</tr>
<tr>
<td>A sea urchin sperm advancing the species at 0.2 mm/s</td>
<td>0.03</td>
</tr>
<tr>
<td>A bacterium, swimming at 0.01 mm/s</td>
<td>0.00001</td>
</tr>
</tbody>
</table>

*Note: High, Int., Low indicate different ranges of Reynolds number.*

from Steve Vogel
Fluid-structure interactions is a very old and complicated subject...

Hui-Neng (638-713)

Hui-Neng, the two monks, and the flag

$Re \sim 10^5$
The debate resumes in 1878

Lord Rayleigh identifies a weak instability of flat slip surfaces (of zero mean circulation) in inviscid fluids (\(Re = \infty\)).

He remarks: “Its bearing upon the flapping of flags and sails will be evident.”

Flapping flags in soap-films -- Zhang, Childress, Libchaber, Sh, Nature 2000
Precipitated new simulations, analyses, and experiments, to reveal the importance of

**flag mass (inertia) modulated by flag elasticity**

Also tension, vortex shedding, viscosity ...

A flexible metal water tunnel flag (to prove a theoretical prediction)
Sh. Vandenberghe, Zhang, *PRL* 2005

---

Rayleigh’s model made full
Alben & Shelley, *PRL* 2009
Drag reduction by self-streamlining of flexible bodies

Studied using experiment, computation, asymptotic analysis

Rigid Bodies: Drag $\sim U^2$
Flexible Bodies: Drag $\sim U^{4/3}$


Same scaling later found for flexible cones...

Photographs by Steven Vogel
In natural erosion, shape and flow also co-evolve…

Wind eroded “Yadan” features in western China

Hoodoos in northern Montana (the Jerusalem Rocks)

Oriented meteorites formed through ablation
AML experiment: clay in flowing water. \( U=60 \text{ cm/s}, \ Re = 10^4 \)

Large time-scale separation of flow and wear

Erosion rate \( \sim \) shear stress

From a sphere, a sharp faceted shape appears. Reminiscent of oriented meteorites

Ristroph, Moore et al, *PNAS* 2012
Cylinders: attracted to self-similar cross-sectional shape with surfaces of nearly constant shear-stress. NOT a smoothing dynamics.

Our math’l model combines bdy layer thry with free-streamline thry.

δ ~ LRe^{-1/2} \sim \sqrt{L} \\
τ \sim \mu U/δ \sim L^{-1/2} \\
\dot{A} \sim τL \sim \sqrt{L} \sim A^{1/4} \\
A(t) \sim A_0(t - t_f)^{4/3}

90 deg = angle of constant shear stress

Self-similar shape w power law scaling matches with experiment.
The transition from low Reynolds number to high

Rotating flapping wing experiment of
Vandenberghe, Zhang, Childress, *JFM '04
VCZ '06, Rosselini & Zhang '06

Simulations of a translating, flapping
“free” wing Alben & Shelley, *PNAS '05

Re \approx 50

\begin{align*}
t &= 16
\end{align*}
Active particles, fluids, structures

Extreme version of fluid-body interactions at small scales:
Fluids with suspended active microstructure –
swimmers (bio and synthetic), motor proteins, biopolymers
Reciprocal coupling between active microstructural dynamics
and large-scale flow – multiscale.

**Active matter:** branch of complex fluids and materials
Many interesting phenomena and applications
“There’s lots of room at the bottom” - Feynman

- Bacterial “turbulence” and low Re mixing
- Microfluidic technology
- Microstructural to macroscopic instabilities
- Transport and mixing by active suspensions
- Cellular biomechanics – transport, cell division

---

Mitotic spindle *Needleman Lab*

Pronuclear motion *Shinar et al 2011*

Sperm swimming in a viscoelastic fluid – *Smith et al, 2009*

---

Dombrowski *et al*

B. subtilis

one and many

---

Takagi *et al*

synthetic swimmers

---

Sanchez *et al 2012*

Streaming nematic of MTs/motor-proteins

---

Shinar *et al 2011*
Biological Swimming at low Reynolds number

In the absence of inertia, swimming must rely of non-reciprocal shape deformations. In nature, several mechanisms for swimming are observed:

- flagellar propulsion (sperm, E coli, B subtilis...)
- ciliary propulsion (Paramecium...)
- body deformation (C elegans...)

In all cases, the swimming particle exerts a propulsive force on the fluid, which is exactly balanced by the viscous drag: to leading order, it creates a force dipole.
Getting good at high-fidelity modeling of single or several swimmers ...

Shape-optimized rotating, driven “swimmers”

\[ U_{z}^{GF} = 0.0326 \]

\[ U_{z}^{OPT} = 0.0812 \]

Ghosh & Fischer, Nano L (2009)

Keaveny, Walker, Sh, Nanoletters ‘13

Self-propelled model bacterium

Cisneros, Cortez, Dombrowski, Goldstein, Kessler, ‘07
Much less understood of many with collective behavior

Dynamics of a roiling bacterial bath

Related to flocking and swarming….

A canonical example of active matter meandering jets and vortices at scales larger and faster than those of single organisms (e.g. *B. subtilis*)

Simulations and analysis of suspensions of self-locomoting rods in a Stokesian fluid reveal roles of geometry, locomotory mechanism, and concentration.

Sh. & Saintillan, *PRL '07, '08, PoF '08, '13, JRSI '12, CRP '13*

Sh. & Hohenegger, *PRE '10*

Also Simha and Ramaswamy 2002; Hernandez-Ortiz *et al* 2005, 2009, Underhill *et al*, Sokolov *et al*., Haines *et al*, and many more…
Basic Model: simple self-propelled rods actuated by surface stress in a Stokes fluid

Saintillan & Shelley '07,'08a&b,'12,'13

\[ |u(x)| \sim \frac{U_0}{|x|^{\frac{2}{3}}} \]
(stresslet)

\[ |u(x)| \sim \frac{U_0}{|x|^{\frac{4}{3}}} \]
(Stokes quadrupole)

\[ |u(x)| \sim \frac{U_0}{|x|^{\frac{2}{3}}} \]
(stresslet)
Mixing of a scalar field by background velocity

\[ \tilde{v} = 0.1 \]

\[ \tilde{v} = 1.0 \]

\[ \tilde{v} = 1.0 \]

Consistent w. Sokolov et al '09 and Hohenegger & Shelley '10
Some things we’ve learned:

Existence and nature of large-scale flow instabilities; mixing properties; effect of propulsion mechanism and body shape; bizarre rheology; …

Beginning to understand:

Hydrodynamic vs. steric interactions
Effects of confinement; interactions with walls
Interactions with other fields – chemical, magnetic, …
Extracting useful work; making pumps?
Collective effects in other media.

Lushi, Goldstein, Sh 2013
Ezhilan, Sh, Saintillan 2013
E. Lushi 2013
Motions driven by microtubule/motor-protein assemblies

Pronuclear migration

Mitotic spindle

Needleman Lab, Harvard

MT-nematic turbulence

Dogic Lab, *Nature* 2012

Synthetic biofluid made of MTs, kinesin clusters, PEG, ATP

Shinar, Mano, Piano, Shelley

*PNAS* 2011
Cytoplasmic Pulling and Pronuclear Motion

w. Tamar Shinar, Miyeko Mana, and Fabio Piano, *PNAS* ’11

Motion of male/female pronuclei (w. minus-end directed cytoplasmic flows) in early development.

**Figure 2** Pronuclear migration in a live *C. elegans* embryo, with GFP fused to β-tubulin. From left to right: pronuclear migration of the male pronucleus (top) and female pronucleus (bottom), pronuclear meeting, centration, rotation. Images courtesy of Miyeko Mana, Department of Biology, NYU.
**Our Model:** Motor proteins pull on MTs and drag fluid:

MTs grow and shrink via polymerization & depolymerization (dynamic instability)

yields MT-length dependent pulling forces

inspired by Kimura & Onami, Dev. Cell 2005
Our model has 5 components
The cytoplasmic flow:

Cytoplasm streams along MTs towards centrosomes
Conclusions

Simple model robustly captures migration, centration, and rotation, and minus-end directed cytoplasmic flow Shinar et al, PNAS ’11

Centration arises through a simple torque instability Fang & Shelley ‘13

Migration via pulling from cytoplasmically-bound motor proteins supported by recent experiments of Kimura & Kimura PNAS ’11

Knock-down of proteins involved in binding dynein to yolk granules significantly reduces streaming of organelles and centration.

Simulations consistent with measured motor protein densities and forces (on the low end, which we may need).

Role of cortical motions and the actin polymer network? Likely involved in earlier stage of female migration.
Dynamics of MT/motor-protein assemblies

Dogic Lab @ Brandeis: synthetic fluids assembled from MTs, kinesin motor complexes, and ATP

Self-assembly of bundles in bulk; merging, sorting, fracturing, …

High concentration on surface: polarity sorting leads to nematic “turbulence”, defect production.

One basic interaction: polarity sorting of anti-aligned MTs

For bundle of \( m \) left-polar MTs, and \( n \) right-polar MTs

Ctr of mass motions: 
\[
\dot{x}_c^L = \frac{2n}{n+m} v, \quad \dot{x}_c^R = -\frac{2m}{n+m} v
\]

Induced active stress: 
\[
\Sigma^a = -\frac{\eta v_w l}{V} \left( x_c^L - x_c^R \right) \frac{mn}{m+n} xx
\]

Motor protein binding bias (towards overlap) yields negative force dipole
Macroscopic Doi-Onsager theory for an surface bound active nematic driven by polarity sorting and active stresses

Kinetic MC provides closure parameters (micro to macro)

Complex, chaotic surface flows, and spontaneous generation and destruction of order \( \pm \frac{1}{2} \) disclination defects

Gao, Blackwell, Glazer, Betterton, & Sh ‘13
Building and analytical and numerical methods for examining
• Centering
• Oscillations
• Self-Assembly

Quantitative Polarized Light Microscopy
Thanks!