TILING REGULAR SHAPES
These convex polygon tiles cover flat space entirely and perfectly, with no overlap and no gaps. These tiles are:

- **MONOHEDRAL** – They consist of only one type of tile.
- **EDGE-TO-EDGE** – The corners of the tiles match up with other corners.
- **REGULAR** – Each tile’s side lengths are all the same, as are its interior angles.

It is easy to find which regular shapes will form a legitimate monohedral, edge-to-edge tiling when we consider that the angles around a point need to add up to exactly 360 degrees in order for the tiles to fit snugly together, with no overlap or gaps.

TILING IRREGULAR PENTAGONS
It’s possible to find an irregular convex pentagon that tiles, but it’s very tricky! In addition to considering whether the angles will sum to 360 degrees, we also need to determine whether the lengths of the sides will allow for edge-to-edge tiling.

In this example, it is possible to find arrangements of angles that result in 360 degrees: 170° + 100° + 90° = 360° and 90° + 90° + 90° + 90° = 360°

However, when we start tiling this pentagon, we find that the side lengths do not allow us to create an edge-to-edge pattern.

**Q** Show how you can divide a regular hexagon into two congruent irregular pentagons that tile the plane.

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**Troublesome Tiles**
People have been studying how to fit shapes together to make toys, floors, walls and art — and to understand the mathematics behind such patterns — for thousands of years.

**Complete the table to discover which of these regular shapes will form a monohedral, edge-to-edge tiling.**

Explain why, if you continue the table beyond a certain number of sides, you’ll find that no other regular polygon can tile the plane.

<table>
<thead>
<tr>
<th>Regular tile shape</th>
<th>△</th>
<th>□</th>
<th>□</th>
<th>□</th>
<th>□</th>
<th>□</th>
<th>□</th>
<th>□</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sides (n)</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Sum of interior angles ((n - 2) \times 180°)</td>
<td>180°</td>
<td>360°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Each interior angle</td>
<td>60°</td>
<td>90°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Is a legitimate tiling possible?</td>
<td><strong>YES</strong></td>
<td><strong>YES</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6 tiles around the point
6 * 60 = 360°

4 tiles around the point
4 * 90 = 360°

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**ANSWER KEY**

There are no monohedral, edge-to-edge, regular tilings for \( n \geq 7 \). Since at least three tiles need to meet at each corner to form 360 degrees, each interior angle of the tile cannot be greater than 120 degrees.

<table>
<thead>
<tr>
<th>Regular tile shape</th>
<th>△</th>
<th>□</th>
<th>□</th>
<th>□</th>
<th>□</th>
<th>□</th>
<th>□</th>
<th>□</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sides ((n))</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Sum of interior angles ((n - 2) \times 180^\circ)</td>
<td>180°</td>
<td>360°</td>
<td>540°</td>
<td>720°</td>
<td>900°</td>
<td>1080°</td>
<td>1260°</td>
<td></td>
</tr>
<tr>
<td>Each interior angle</td>
<td>60°</td>
<td>90°</td>
<td>108°</td>
<td>120°</td>
<td>128.57°</td>
<td>135°</td>
<td>140°</td>
<td></td>
</tr>
<tr>
<td>Is a legitimate tiling possible?</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td></td>
</tr>
</tbody>
</table>

**USING HEXAGONS TO FIND IRREGULAR PENTAGON TILINGS:**

1. Regular hexagon tiles will easily tile the plane, edge-to-edge.
2. Draw a line from one side to the opposite side to create two congruent pentagons. As long as the line passes through the center of the hexagon, the two pentagons will be congruent.
3. The irregular pentagons can now be tiled. Some tiles may need to be flipped over to ensure edge-to-edge tiling.

**BONUS QUESTIONS**

Only eight types of irregular pentagons tile edge-to-edge. Research one and share your findings.

Additional tilings can be found when we relax the edge-to-edge restriction. Divide a hexagon into three congruent pentagons to show how they can tile the plane without being edge-to-edge.