Jamming in biological tissues

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Are you a solid or a fluid?
early embryonic tissues are viscoelastic
example: zebrafish

timescale \sim \text{seconds}

timescale \sim \text{hours}

E.-M. Schoetz thesis

Schoetz, Lanio, Talbot, MLM
J. R. Soc. Interface (2013)
Cultured lung epithelial layer solidify over time

Day 3
Day 6
Day 10

Cilia
Nuclei
Cytoplasm

Primary Human Bronchial Epithelial Cells (HBECs)

Liquid-solid transitions also occur in cancer cell lines

- **model for metastatic** breast cancer cells
- **model for normal** breast cells
- **model for malignant** breast cancer cells


Käs lab, Leipzig University
Can we understand how cells control these fluid-solid transitions in dense tissues?
What happens when you have a lot of strongly interacting objects at high densities?

Cells (Schoetz Lab UCSD)  
Mayonnaise
What happens when you have a lot of strongly interacting objects at high densities?

Cells (Schoetz Lab UCSD)

Dense emulsion (Thijssen)
What happens when you have a lot of strongly interacting objects at high densities?

not just mayonnaise: also polymers, foams, grain silos, earthquake faults, bulk metallic glasses
What happens at high densities?

What happens at high densities?

What happens at high densities?

What happens at high densities?


[Diagram showing temperature and density relationship with jamming and glass transition points.]
What happens at high densities?

What happens at high densities?


Trappe et al, Nature 411, 772-775 (2001)
How to quantify whether a system is near a ordered fluid-to-solid transition

This is straightforward. Identify an order parameter, such as bond orientational order.

Use the order parameter to search for coexistence in a first-order phase transition.

Computational model for water, Malolepsza and Keyes, JCTC 2016
How to quantify whether a system is near a disordered fluid-to-solid transition

No simple structural order parameter

Look at the dynamics instead
How to quantify whether a system is near a disordered fluid-to-solid transition

More dynamics:
Caging behavior measured by non-gaussian parameter

Kob et al, PRL 79 15 2827 (1997)
How to quantify whether a system is near a disordered fluid-to-solid transition

Kob et al., PRL 79 15 2827 (1997)


Or identify dynamical heterogeneities using a 4-point correlation function

More dynamics: Caging behavior measured by non-gaussian parameter

\[ G_4((r - r'); (t - t')) = \rho(r', t') \rho(r', t) \rho(r, t') \rho(r, t) > \]
How to quantify whether a system is near a disordered fluid-to-solid transition

Look for a scaling collapse in control parameter:
  coordination number $Z$ or density $\rho$
Does this really happen in biological tissues?

Schoetz, Lario, Talbot, MLM
J. R. Soc. Interface 10(89), 20130726 (2013)
Does this really happen in biological tissues?

Primary Human Bronchial Epithelial Cells (HBECs) exhibit dynamical heterogeneities.
Does this really happen in biological tissues?

Day 3

Day 6

Day 10

Cilia

Nuclei

Cytoplasm

Lamina propria
(loose connective tissue)

Modeling cells using “active particles”

- self propelled particle models
- speed $v_0$ and persistence length governed by rotational noise $D_r$
- add short-range repulsive (and maybe attractive) interactions to mimic cells
- some models also have alignment
- really interesting non-equilibrium behavior (flocking, motility induced phase separation)

Glass transition in self-propelled particle models is identical to adhesive colloids*

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Glass transition in self-propelled particle models is identical to adhesive colloids.*

Glass transition in self-propelled particle models is identical to adhesive colloids*.

*almost: \( \phi_J \) changes a little with the persistence time
Proposed jamming phase diagram for biological tissues

Vector
near jamming transition

Temperature/adhesion

applied force (shear stress)

1/density


Differentiation **86** (2013)
Proposed jamming phase diagram for biological tissues

Packing fraction = 1
In many tissues, a density-driven transition is impossible
e.g. confluent tissues where there are no gaps between cells
and the packing fraction is one
Is there some analogue to the jamming transition for
confluent tissues?
Vertex models for tissues

- Developed about 15 years ago
- Good agreement with experimentally observed cell shapes
- Explain/predict mechanically stable cell shapes and statistical properties

Farhadifar et al, Current Biology (2007)
MLM et al, PNAS (2010)
Staple et al EPJE 33 (2) 117 (2010)
Chiou et al PLOS Comp Bio 8 (5) e1002512 (2012)
Vertex model equations

\[ E_{cell} = k_A (A - A_0)^2 + k_P (P - P_0)^2 \]
\[ = k_A (A - A_0)^2 + k_P (P^2 - 2P_0P + P_0^2) \]

A = area, P = perimeter

3D Incompressibility + resistance to height fluctuations
Interfacial tension: adhesion and cortical tension
acto-myosin contractility

\[ \varepsilon = \frac{1}{\beta A_0} \sum_i^N E_i = \sum_i \left[ (a_i - 1)^2 + \frac{(p_i - p_0)^2}{r} \right] \]

Non-dimensionalized mechanical energy
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Non-dimensionalized mechanical energy

Our model has two parameters:

- \( p_0 \) = target shape index: target perimeter /square root of target area: increases with increasing adhesion
- \( r \) = ratio between bulk stiffness and interface stiffness
Rearrangements and migration in epithelial sheets must occur via T-I transitions

Bi, Lopez, Schwarz, MLM Soft Matter (2014)
Work required to execute a T-1 transition:

Shrink edge before T-1  
At T-1  
Grow edge after T-1

Bi, Lopez, Schwarz, MLM Soft Matter (2014)
Work required to execute a T-1 transition:

1. Shrink edge before T-1
2. At T-1
3. Grow edge after T-1

Figure showing the transition with graphs and a diagram of the process.

Bi, Lopez, Schwarz, MLM Soft Matter (2014)
Average energy barrier height vanishes at $p_0^* \sim 3.81$

Bi, Lopez, Schwarz, MLM Nature Physics (2015)
A critical rigidity transition controlled by $p_0$:

$$r \frac{\Delta \varepsilon}{|p_0 - p_0^*|} = \beta f_\pm \left( \frac{r}{|p_0 - p_0^*|} \right)$$

**Graph**

- **Case**: $p_0 < p_0^*$
- **Case**: $p_0 > p_0^*$

- $p_0^* = 3.813 \pm 0.005$
- $\Delta = 4.0 \pm 0.4$
- $\beta = 1.0 \pm 0.2$

**Equation**

$$z = \frac{r}{|p_0 - p_0^*|}$$

*Bi, Lopez, Schwarz, MLM Nature Physics (2015)*
New rigidity phase diagram for biological tissues

New rigidity phase diagram for biological tissues

Particulate matter: axis is \( \frac{1}{\text{adhesion}} \)
New rigidity phase diagram for biological tissues

Particulate matter: axis is $1/\text{adhesion}$

more adhesion means more gelation means more solid-like
New rigidity phase diagram for biological tissues

Particulate matter: axis is 1/adhesion
more adhesion means more gelation means more solid-like

Confluent tissues axis is adhesion (or larger preferred perimeter)
more adhesion means larger cell perimeters means more degrees of freedom means more liquid-like

Opposite for confluent tissues!
New rigidity phase diagram for biological tissues

Particulate matter:
- axis is $1/\text{adhesion}$
- more adhesion means more gelation means more solid-like

Confluent tissues:
- axis is adhesion (or larger preferred perimeter)
- more adhesion means larger cell perimeters means more degrees of freedom means more liquid-like

First prediction: more adhesion can actually lead to unjamming in a tissue

Opposite for confluent tissues!
New order parameter: shape index

Recall: $p_0$ is a model parameter, which is the target perimeter-to-area ratio.

Define the observable shape index $p$:

$$p_i = \frac{P_i}{\sqrt{A_i}}; \quad \bar{p} = \text{median}\{p_i\}$$

Simulations of vertex model suggest $\bar{p}$ is an order parameter

New order parameter: shape index

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Simulations of vertex model suggest $\bar{p}$ is an order parameter.

Prediction:

A jammed tissue should have a $\bar{p}$ that is precisely 3.81. As a tissue becomes more unjammed, its $\bar{p}$ should increase above 3.81.

Cultured lung epithelial layer becomes solid-like over time

Shape index $p$ approaches precisely the predicted value at jamming.

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Shape index p approaches precisely the predicted value at jamming
Shape index $p$ approaches precisely the predicted value at jamming.
Shape index p approaches precisely the predicted value at jamming.

What happens when you add active cell motility?

2 parameters like self-propelled particles:

- particle speed $v_0$
- persistence length governed by white rotational noise $D_r$
Effect of finite cell motility?

Solid/liquid phases determined by diffusion constant

increasing cortical tension

increasing cell-cell adhesion

Bi, Yang, Marchetti, MLM, PRX (2016)
Effect of finite cell motility?

Solid/liquid phases determined by diffusion constant

increasing cortical tension
increasing cell-cell adhesion

Bi, Yang, Marchetti, MLM, PRX (2016)

The figure shows a graph with two axes, labeled $p_0$ and $V_0$, with corresponding values indicating the phase transition between solid and liquid. The graph illustrates how the system's behavior changes with increasing $p_0$ and $V_0$, leading to different phases.
Does the shape index still indicate a fluid to solid transition?

\[ p_0 = \langle \text{perimeter}/\sqrt{\text{area}} \rangle \]
Does the shape index still indicate a fluid to solid transition?

\[ p = 3.813 \]

- \( p > 3.813 \) indicates Diffusive behavior.
- \( p < 3.813 \) indicates Non-diffusive behavior.

shape order parameter: \( p = \langle \text{perimeter}/\sqrt{\text{area}} \rangle \)
New rigidity phase diagram for biological tissues

Motility $v_0$

Cell-cell adhesion $p_0$

Persistence $D_r$

Solid-like


Bi, Yang, Marchetti, MLM, PRX (2016)
New rigidity phase diagram for biological tissues

Motility $v_0$

Cell-cell adhesion $p_0$

Persistence $1/D_r$

Solid-like

Bi, Yang, Marchetti, MLM, PRX (2016)
Work in progress
The direction $\hat{n}_i$ diffuses on the unit sphere with rate $D_r$

$$\dot{r}_i = v_0 \hat{n}_i + \mu F_i$$

$$E = \sum_i K_V (V_i - V_0)^2 + K_S (S_i - S_0)^2$$

$$F_i = - \frac{\partial E}{\partial r_i}$$

$$s_0 = \frac{S_0}{V_0^{2/3}}$$
$s_0 = 5.3$
$v_0 = 0.1$
$D_r = 1$
\[ s_0 = 5.5 \]
\[ v_0 = 0.1 \]
\[ D_r = 1 \]
Fluid-solid transition identified in mean-squared displacements.

Transition point in 3D: $s_0 = 5.41$
Still a second order critical phase transition with a structural order parameter
What is the origin of the rigidity transition?

What sets $3.81$ (2D) or $5.41$ (3D)?

Why are these values so robust to perturbing the model?

Is there an index theorem associated with this?
Onset of jamming in particulate systems: Maxwell criterion or Calladine index theorem

1. On the Calculation of the Equilibrium and Stiffness of Frames.

By J. Clerk Maxwell, F.R.S., Professor of Natural Philosophy in King’s College, London*. Phil. Mag 1864

BUCKMINSTER FULLER’S “TENSEGRITY” STRUCTURES AND CLERK MAXWELL’S RULES FOR THE CONSTRUCTION OF STIFF FRAMES

C. R. Calladine
University of Cambridge, Department of Engineering, Trumpington Street, Cambridge CB2 1PZ, England


Topological Boundary Modes in Isostatic Lattices

C. L. Kane and T. C. Lubensky
Dept. of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104

Nature Physics 2013

\[ N_{dof} - N_0 = N_{sp} - N_{ss} \]

Degrees of freedom
Infinitesimal zero modes
“springs” or constraints
States of self-stress (e.g. bonds under tension)
Onset of jamming in particulate systems: Maxwell criterion or Calladine index theorem

Rigidity should occur when there are only trivial zero modes (global rotation, translation)

Explains onset of jamming in particulate systems

\[ N_{dof} - N_0 = N_{sp} - N_{ss} \]

- Degrees of freedom
- Infinitesimal zero modes
- "springs" or constraints
- States of self-stress (e.g. bonds under tension)
This theorem can be generalized to the vertex model

Infinitesimal zero mode $\tilde{z}$ defined by:

$$\forall i : \quad \frac{\partial A_i}{\partial r^i} \cdot \tilde{z} = 0 \quad \land \quad \frac{\partial P_i}{\partial r^i} \cdot \tilde{z} = 0$$

These infinitesimal zero modes form a $N_0$-dimensional vector space.

$$N_{dof} - N_0 = N_{sp} - N_{ss}$$

But UNLIKE particle jamming, these $N_0$ infinitesimal zero modes are not necessarily zero modes of the dynamical matrix $D$ (D is the hessian of the energy function)

$$\tilde{z} \cdot D \cdot \tilde{z} = \sum_i \left[ \frac{\partial^2 E}{\partial A_i^2} \left( \frac{\partial A_i}{\partial r^i} \cdot \tilde{z} \right) \left( \frac{\partial A_i}{\partial r^i} \cdot \tilde{z} \right) + \frac{\partial^2 E}{\partial P_i^2} \left( \frac{\partial P_i}{\partial r^i} \cdot \tilde{z} \right) \left( \frac{\partial P_i}{\partial r^i} \cdot \tilde{z} \right) + \frac{\partial E}{\partial A_i} \tilde{z} \cdot \frac{\partial^2 A_i}{\partial r^2} \cdot \tilde{z} + \frac{\partial E}{\partial P_i} \tilde{z} \cdot \frac{\partial^2 P_i}{\partial r^2} \cdot \tilde{z} \right]$$
The index theorem doesn’t seem to explain onset of rigidity in vertex model.

\[
N_{dof} - N_0 = N_{sp} - N_{ss}
\]
\[
N_{dof} = 4N_{\text{cells}}
\]
\[
N_{sp} = N_{\text{cells}}
\]

**solid regime:**
- \(N_{ss} = 1\)
- \(N_0 = 3N_{\text{cells}} + 1\)
- \(N_D = 0\)

**fluid regime:**
- \(N_{ss} = 0\)
- \(N_0 = 3N_{\text{cells}}\)
- \(N_D = 3N_{\text{cells}}\)

\[ N_{\text{cells}} = 100 \]
Simple example:

\[ N_{dof} - N_0 = N_{sp} - N_{ss} \]

Floppy network:

\[ N_{ss} = 0 \quad N_0 = 1 \quad N_D = 1 \]

\[ l = l_0 \]

\[ d < 3l_0 \]

Rigid network:

\[ N_{ss} = 1 \quad N_0 = 2 \quad N_D = 0 \]

\[ l > l_0 \]

\[ d > 3l_0 \]
What might work: geometric constraint
Fixed area, $p_0$ only control parameter

(a)

(Bi, Lopez, Schwarz, MLM Nat Phys (2015))
What might work: geometric constraint

Fixed area, $p_0$ only control parameter

(a) Geometric configurations

(b) Graph showing $\epsilon - \epsilon(\ell = 0)$ vs $\ell$ for increasing $p_0$

(c) Graph showing $\Delta \omega$ vs $p_0$


Transition point $p_0^* = 3.813 = \text{perimeter of regular pentagon with unit area}$
One application: can we find new signatures of cancer invasiveness?

primary tumor cells from patients with breast and cervical cancer

Franziska Wetzel\textsuperscript{1}, Dapeng Bi\textsuperscript{3}, Anatol Fritsch\textsuperscript{1}, Steffen Grosser\textsuperscript{1}, Linda Oswald\textsuperscript{1}, Steve Pawlizak\textsuperscript{1}, Lars-Christian Horn\textsuperscript{1}, Michael Höckel\textsuperscript{1}, Susanne Briest\textsuperscript{1}, Cristina Marchetti\textsuperscript{2}, Lisa Manning\textsuperscript{2}, Josef Käs\textsuperscript{1},
\textsuperscript{1}Leipzig University, \textsuperscript{2}Syracuse University, \textsuperscript{3}Rockefeller University
Cancer cells are softer, yet tumors are more rigid…
Cancer cells are softer, yet tumors are more rigid…
What happens to rigidity transition when there is a broad distribution of cell stiffnesses?
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What happens to rigidity transition when there is a broad distribution of cell stiffnesses?
What happens to rigidity transition when there is a broad distribution of cell stiffnesses?
\[ F_r = \text{fraction of cells with } p_0 \leq 3.81 \]
A tumor can be rigid even when more than 50% of its cells are soft.
A tumor can be rigid even when more than 50% of its cells are soft.

Generalized Rigidity transition
0.475 ± 0.005
Also naturally gives rise to “multi-cellular streaming” seen in some cancer tumors.
Spontaneous organization of soft cells into quasi-1D streams
Spontaneous organization of soft cells into quasi-1D streams
Conclusions

• Many biological tissues are apparently close to a glass or jamming transition
• The vertex model for confluent tissues exhibits a novel type of rigidity transition
  • instead of density, control parameter is target shape index $p_0$
  • structural order parameter: observed perimeter-to-area ratio
• A new SPV model predicts that the rigidity transition controls a line of glass transitions as a function of speed and persistence
• In heterogeneous systems, rigidity controlled by the fraction of soft cells (more than 50% can be soft and the tissue is still rigid)
• 3D SPV model exhibits same transition, with possible connection to particulate matter
Thanks so much for your attention!

Collaborators:
• Max Dapeng Bi (Rockefeller), Matthias Merkel, Xingbo Yang, Cristina Marchetti, Jen Schwarz, Jorge Lopez (SU)
• Jin-Ah Park, Jae Hun Kim, Jen Mitchell, Jeff Fredberg et al (Harvard School of Public Health)
• Kaes Lab (Leipzig University)

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• Gordon and Betty Moore Foundation
• Soft Interfaces IGERT (DGE-1068780)
Two types of zero modes

- Infinitesimal zero modes
- $N_0$ of them
- Zero modes of the dynamical matrix $D$
Two types of zero modes

- Infinitesimal zero modes
- \(N_0\) of them
- Zero modes of the dynamical matrix \(D\)

Rigidity should occur when there are only trivial zero modes (global rotation, translation).

Explains onset of jamming in particulate systems.
future directions

Physics:
• Is the rigidity transition topologically protected? Does it satisfy an index theorem?
• Fluid hyperuniform states with photonic band gaps?
• Can pressure, stress, surface tension be generalized to these systems?

Biology:
• Check validity of structural order parameter in a wider range of tissues and make predictions for traction forces
• What is the relationship of between the epithelial-mesenchymal transition in cancer and the fluid/solid transition? What is the impact on metastasis?
• Can cells prevent T1 transitions (or delay them) and what is impact on tissue structure and rheology (rosettes?)
Effect of finite cell motility

- Voronoi tessellation and shape energy:

\[ E_{tissue} = \sum_i \left[ (a_i - 1)^2 + r(p_i - p_0)^2 \right] \]

- Add motility and persistence:

\[
\frac{d\mathbf{r}_i}{dt} = v_0 \hat{n}_i - \frac{\partial E_{tissue}}{\partial \mathbf{r}_i}
\]

\[
\frac{d\theta_i}{dt} = \sqrt{2D_r} \eta_i(t) \quad \langle \eta_i(t)\eta_j(t') \rangle = \delta_{ij}\delta(t-t')
\]

\[ \hat{n}_i = (\cos \theta_i, \sin \theta_i) \]

\[ A = \text{area} \]

\[ P = \text{perimeter} \]

related model: Li & Sun Biophys. J. 2014
Effect of finite cell motility

- Voronoi tessellation and shape energy:
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- Add Motility and persistence:
  \[
  \frac{d\vec{r}_i}{dt} = v_0\hat{n}_i - \frac{\partial E_{tissue}}{\partial \vec{r}_i}
  \]
  \[
  \frac{d\theta_i}{dt} = \sqrt{2D_r}\eta_i(t) \quad \langle \eta_i(t)\eta_j(t') \rangle = \delta_{ij}\delta(t-t')
  \]

Force between cell is multicellular and cannot be expressed as a pairwise interaction

related model: Li & Sun Biophys. J. 2014
Related to classical jamming?

Sphere packing at jamming

Voronoi tessellation

<table>
<thead>
<tr>
<th>Parameter $\chi$</th>
<th>$N$</th>
<th>$S$</th>
<th>$\tilde{V}$</th>
<th>$S/V^{(d-1)/d}$</th>
<th>$A$</th>
<th>$\cos \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appx. power $\gamma$</td>
<td>0.7</td>
<td>1.0</td>
<td>0.75</td>
<td>1.0</td>
<td>0.75</td>
<td>0.33</td>
</tr>
<tr>
<td>$\chi_J$, $d = 2$, large</td>
<td>6</td>
<td>3.00(5)</td>
<td>0.03(1)</td>
<td>3.73(8)</td>
<td>1.22(1)</td>
<td>0.56(7)</td>
</tr>
<tr>
<td>$\chi_J$, $d = 2$, small</td>
<td>6</td>
<td>2.27(5)</td>
<td>0.04(7)</td>
<td>3.82(7)</td>
<td>1.30(6)</td>
<td>0.55(9)</td>
</tr>
<tr>
<td>$\chi_J$, $d = 2$, all</td>
<td>6</td>
<td>2.64(0)</td>
<td>0.29(5)</td>
<td>3.78(2)</td>
<td>1.26(4)</td>
<td>0.56(3)</td>
</tr>
<tr>
<td>$\chi_J$, $d = 3$</td>
<td>14.29</td>
<td>5.3(8)</td>
<td>0.03(8)</td>
<td><strong>5.3(8)</strong></td>
<td>1.32(2)</td>
<td><strong>0.42(9)</strong></td>
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<tr>
<td>$\chi_J$, $d = 4$</td>
<td>32.74</td>
<td>6.8(7)</td>
<td>0.03(6)</td>
<td>6.8(8)</td>
<td>1.3(7)</td>
<td>0.38(5)</td>
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<tr>
<td>$\chi_J$, $d = 5$</td>
<td>74.62</td>
<td>8.2(6)</td>
<td>0.03(4)</td>
<td>8.2(6)</td>
<td>1.4(1)</td>
<td>0.35(0)</td>
</tr>
</tbody>
</table>

Why 3.81?
Fixed area, $p_0$ only control parameter

(a) 

(b) 

$\epsilon - \epsilon(\ell = 0)$ 

Increasing $p_0$

(c) 

$\Delta \omega$

$10^0$ $10^{-2}$ $10^{-4}$ $10^{-6}$

3 3.5 4

Why 3.81?
Fixed area, $p_0$ only control parameter

(a) Bi, Lopez, Schwarz, MLM Nat Phys (2015)

(b) Transition point $p_0^* = 3.813 = \text{perimeter of regular pentagon with unit area}$

(c) $\Delta \omega$
Rosette formation

Drosophila (fruit fly) embryonic epithelia during convergent extension

Existing vertex models: 3-fold coordinated vertices and 4-fold transition points

What about higher order transitions?

Kasza and Zallen, PNAS 2014
Intra-tissue tension higher in asthmatic tissues

Unjammed tissues support higher tensile stresses (not less)
Consistent with idea that adhesive interactions are stronger in the unjammed tissues
Instantaneous displacements at low Dr follow collective normal modes as $1/\omega^2$.
p a good order parameter at all values of Dr
A critical rigidity transition controlled by $p_0$

Average energy barrier height obeys the scaling function

$$r \Delta \varepsilon = |p_0 - p_0^*|^{\beta} f_{\pm} \left( \frac{r}{|p_0 - p_0^*| \Delta} \right)$$

### Comparison to other rigidity transitions

<table>
<thead>
<tr>
<th>Order Parameter</th>
<th>Rigidity Transition, Confluent Tissues</th>
<th>Jamming Transition (2D)</th>
<th>Cross-linked Fiber Networks (2D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy barrier height</td>
<td>$</td>
<td>p_0 - p_0^*</td>
<td>$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\beta = 1$</td>
<td>$\beta = 1.65$</td>
<td>$\beta = 1.4$</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>$\Delta = 4$</td>
<td>$\Delta = 1.2$</td>
<td>$\Delta = 3$</td>
</tr>
</tbody>
</table>

**Exponents**

Jamming: Olsson & Teitel PRL 2007  
Das, Quint & Schwarz 2012  
Glass transition in self-propelled particle models is identical to adhesive colloids.

\[ \phi_1 \rightarrow \phi_2 \]

\[ \text{vanishing persistence (colloids)} \]

\[ \text{particle speed} \]

\[ \text{density} \]

Berthier PRL 2014
Fily, Henkes & Marchetti Soft Matter 2014
Glass transition in self-propelled particle models is identical to adhesive colloids*

![Graph showing the relationship between density and particle speed with critical densities \( \phi_{c1} \) and \( \phi_{c2} \). The graph indicates a phase change from liquid to solid with vanishing persistence (colloids).]
Glass transition in self-propelled particle models is identical to adhesive colloids*

\[ \phi \]

Berthier PRL 2014
Fily, Henkes & Marchetti Soft Matter 2014
Glass transition in self-propelled particle models is identical to adhesive colloids*

*almost: $\phi_c$ changes a little with the persistence time; the activity generates an effective adhesion
A tumor can be rigid even when more than 50% of its cells are soft!
Rigidity transition with heterogeneity
Dynamical heterogeneities

A colloidal glass.

Displacement profile in simulation of a 2-d glass former. Berthier PRL 2011
Four point correlation functions: captures “swirls” or “dynamical heterogeneities”

\[ G_4((r - r'); (t - t')) = \langle \rho(r', t')\rho(r', t)\rho(r, t')\rho(r, t) \rangle \]
Four point correlation functions:
captures “swirls” or “dynamical heterogeneities”

\[ G_4((r - r'); (t - t')) = < \rho(r', t')\rho(r', t)\rho(r, t')\rho(r, t) > \]

\[ Q_l(l, \tau) = \frac{1}{N} \sum_{i=1}^{N} w_i, \quad w_i = \begin{cases} 
1, & \text{overlap} > l \\
0, & \text{overlap} < l 
\end{cases} \]
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\[ l = \frac{1}{2} d \]
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\[ Q_l(l, \tau) = \frac{1}{N} \sum_{i=1}^{N} w_i, \quad w_i = \begin{cases} 1, & \text{overlap} > l \\ 0, & \text{overlap} < l \end{cases} \]

\[ l = \frac{1}{2} d \]

\[ w_i = 1, \quad t + t' \]

\[ t \]

\[ \tau \]

\[ w_i \]

\[ 1 \]

\[ 0 \]
Four point correlation functions: captures “swirls” or “dynamical heterogeneities”

\[ G_4((r - r'); (t - t')) = < \rho(r', t')\rho(r', t)\rho(r, t')\rho(r, t) > \]

\[ Q_t(l, \tau) = \frac{1}{N} \sum_{i=1}^{N} w_i, \quad w_i = \begin{cases} 
1, & \text{overlap} > l \\
0, & \text{overlap} < l 
\end{cases} \]

\[ l = \frac{1}{2} d \]
Four point correlation functions: captures “swirls” or “dynamical heterogeneities”

\[ G_4((r - r'); (t - t')) = \langle \rho(r', t')\rho(r', t)\rho(r, t')\rho(r, t) \rangle \]

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\[ m = \sum_i S_i^{\alpha \beta} \]

\[ \chi_4 = \left( \langle q^2 \rangle - \langle q \rangle^2 \right) \]

\[ \chi = \frac{1}{N} \left( \langle m^2 \rangle - \langle m \rangle^2 \right) \]

\[ m = \sum_i S_i \]

\[ wi = 1, \quad \text{overlap} > l \]

\[ wi = 0, \quad \text{overlap} < l \]
Four point correlation functions

\[ Q_i(l, \tau) = \frac{1}{N} \sum_{i=1}^{N} w_i, \]

\[ Q(l, \tau) = \langle Q_i(l, \tau) \rangle. \]

<table>
<thead>
<tr>
<th>overlap threshold, ( l/d_s )</th>
<th>0.03</th>
<th>0.06</th>
<th>0.08</th>
<th>0.11</th>
<th>0.14</th>
<th>0.18</th>
<th>0.23</th>
<th>0.28</th>
<th>0.32</th>
<th>0.38</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q(l, \tau)</td>
<td>0.42</td>
<td>0.50</td>
<td>0.56</td>
<td>0.84</td>
<td>1.13</td>
<td>1.41</td>
<td>2.81</td>
<td>4.22</td>
<td>5.63</td>
<td></td>
</tr>
</tbody>
</table>
Four point correlation functions

\[ Q_i(l, \tau) = \frac{1}{N} \sum_{i=1}^{N} w_i, \]

\[ Q(l, \tau) = \langle Q_i(l, \tau) \rangle, \]

\[ \chi_4(l, \tau) = N[\langle Q_i(l, \tau)^2 \rangle - \langle Q_i(l, \tau) \rangle^2] \]

Abate and Durian PRE 2007
Self-stress $\vec{s}$ defined by:

$$\sum_i \left( s_{2i} \frac{\partial A_i}{\partial \vec{r}} + s_{2i+1} \frac{\partial P_i}{\partial \vec{r}} \right) = 0$$
Residual stresses create rigidity in the vertex model.

**infinitesimal zero modes:**
\[ N_{dof} - N_0 = N_{sp} - N_{ss} \]
\[ N_{dof} = 4N_{\text{cells}} \]
\[ N_{sp} = N_{\text{cells}} \]

(here: case without the area term)

**fluid regime:**
\[ N_{ss} = 0 \quad N_0 = 3N_{\text{cells}} \]

**solid regime:**
\[ N_{ss} = 1 \quad N_0 = 3N_{\text{cells}} + 1 \]
Residual stresses create rigidity in the vertex model.

infinitesimal zero modes:

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(here: case without the area term)

solid regime:
\[ N_{ss} = 1 \]
\[ N_0 = 3N_{\text{cells}} + 1 \]

fluid regime:
\[ N_{ss} = 0 \]
\[ N_0 = 3N_{\text{cells}} \]
Residual stresses create rigidity in the vertex model.

infinitesimal zero modes:
\[ N_{dof} - N_0 = N_{sp} - N_{ss} \]
\[ N_{dof} = 4N_{cells} \]
\[ N_{sp} = 2N_{cells} \]

**fluid regime:**
\[ N_{ss} = 1 \]
\[ N_0 = 2N_{cells} + 1 \]

**solid regime:**
\[ N_{ss} = 2 \]
\[ N_0 = 2N_{cells} + 2 \]

\( p_0 = 3.7 \quad N_{cells} = 100 \quad p_0 = 4.0 \)
Residual stresses create rigidity in the vertex model.

**infinitesimal zero modes:**

\[ N_{dof} - N_0 = N_{sp} - N_{ss} \]

\[ N_{dof} = 4N_{\text{cells}} \]

\[ N_{sp} = 2N_{\text{cells}} \]

**solid regime:**

\[ N_{ss} = 2 \]

\[ N_0 = 2N_{\text{cells}} + 2 \]

**fluid regime:**

\[ N_{ss} = 1 \]

\[ N_0 = 2N_{\text{cells}} + 1 \]

\[ N_{\text{cells}} = 100 \]
Residual stresses can create rigidity.

**Infinitesimal zero modes:**

\[ N_{dof} - N_0 = N_{sp} - N_{ss} \]

- \( N_{dof} = 4 \)
- \( N_{sp} = 3 \)
- \( N_{ss} = 0 \)
- \( N_0 = 1 \)

**Floppy network**

- \( d < 3l_0 \)

**Rigid network**

- \( d = 3l_0 \)
- \( d > 3l_0 \)
Residual stresses can create rigidity.

infinitesimal zero modes:

\[ N_{dof} - N_0 = N_{sp} - N_{ss} \]

- \( N_{dof} = 4 \)
- \( N_{sp} = 3 \)

- \( N_{ss} = 0 \)
- \( N_0 = 1 \)

Floppy network

- \( d < 3l_0 \)

Rigid network

- \( d > 3l_0 \)

- \( l > l_0 \)
Residual stresses create rigidity in the vertex model.

(in here: case without the area term)

infinitesimal zero modes:

\[ N_{dof} - N_0 = N_{sp} - N_{ss} \]

\[ N_{dof} = 4N_{cells} \]

\[ N_{sp} = N_{cells} \]

fluid regime:

\[ p_i = p_0 \]

\[ N_{ss} = 0 \]

\[ N_0 = 3N_{cells} \]

solid regime:

\[ p_i > p_0 \]

\[ N_{ss} = 1 \]

\[ N_0 = 3N_{cells} + 1 \]
Residual stresses create rigidity in the vertex model.

(Here: case without the area term)

Infinitesimal zero modes:

\[ N_{dof} - N_0 = N_{sp} - N_{ss} \]
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**Fluid regime:**
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\[ N_{ss} = 1 \]
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