The possible spin liquid phase of J1-J2 Heisenberg model on the triangular lattice

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Outline

- Introduction to quantum spin liquid (QSL) state.
- DMRG study of Heisenberg J1-J2 model on the triangular lattice.
- Phase diagram and ground state properties at J2=0.1 (possibly SL state) 120 degree
- Conclusion and outlook
Searching for quantum spin liquid (QSL) state

1973 P.W. Anderson propose a short range RVB state as the ground state of the Heisenberg triangular lattice.

1988 better variational wave function suggest a long range magnetic order state by Huse and Elser.

2012 $\mathbb{Z}_2$ SL state find on Heisenberg Kagome lattice by Yan, White and Huse.

$H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j$

QSL does not break any symmetries
No magnetic order, no valence bond order, no chiral order
Introduction to the Hamiltonian and lattice cylinder geometry

Frustrate more on the triangular lattice by adding J2 terms.

\[ H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i \cdot S_j \]

Take J1=1, J2>0

Antiferromagnetic case

Classical phase diagram

At J2>1/8, spin wave theory suggest a two sub-lattice collinear state by chubukov 1992
Density matrix renormalization group

\[ |\psi > = \sum_{i=1}^{N_A} \sum_{j=1}^{N_B} \psi_{ij} |i > \otimes |j > \]

\[ \rho_{ii'} = \sum_j \psi_{ij}^* \psi_{i'j} \]

\[ \rho = \sum_\alpha w_\alpha |v_\alpha > < v_\alpha | \]

Read the whole spectrum

Sweep process

DMRG on a cylinder
Map the 2D cylinder into a 1D chain.
Determine the phase transition with $J_2$ gradient on a single cylinder

YC6 cylinder without $U(1)$ symmetry

1. $0 < J_2 < 0.06$, 120 degree Neel order
2. $0.06 < J_2 < 0.15$, SL without magnetic order.
3. $J_2 > 0.15$, Neel order on square lattice (collinear order)
Introduction to even/odd sectors in RVB state

Two leg ladder RVB picture

(a) even sector
vertical line cut even number of bonds

(b) odd sector
vertical line cut odd number of bonds

Boundary condition determine the two different state.
Sometimes even sector state could have odd sector state in the center with end spinons.
Even/Odd topological sectors on YC6 at $J_2=0.1$

Even sector

$E_2 = -0.51485$

$\Delta E = 0.0061$ per-spin

Even sector has bond fluctuation in center.

Odd sector

$E_1 = -0.52096$

Ground state

Uniform bonds in all the directions around -0.18.

All the bonds shown here subtract -0.18.
Spin and bond response

Spin correlation length is 1.88

Bond response by strengthen the center vertical bond.

Both order looks short ranged
Determine the phase transition between SL and collinear State

SL state has lower energy for $J_2 < 0.16$. Collinear state is favored for larger $J_2$. 

$J_2 = 0.25$, Collinear state
Even/odd sectors on YC8 cylinder at $J_2 = 0.1$

Even sector has higher energy with bond variance $E = -0.5179(2)$

Odd sector is ground state $E = -0.51954(2)$

Anisotropic bonds
- Vertical bonds $\sim -0.225$
- Diagonal bonds $\sim -0.155$

Energy difference $\Delta E = 0.0016$, 
For topological SL state, energy splitting between even and odd topological sectors decrease exponentially with the cylinder width.

\[ \Delta E \sim e^{-\frac{L_y}{\xi_s}} \]

\( \xi_s \) is a correlation length

Correlation length is 1.69

Ly is the cylinder width.
Dimerization effect on odd cylinders

On odd cylinders, such as YC(2n+1), XC(4n+2). There are dimerized effect.

Alternating strong and weak bonds along each column.

Solid bonds are stronger than average. Dashed bonds are weaker.

Stronger zigzag bonds, Weaker zigzag bonds

Alternating strong and weak bonds along each column.
Spin gaps at $J_2=0.1$

YC6 triplet gap = 0.3668
singlet gap = 0.384

YC5 triplet gap = 0.289
Conclusion and outlook

- Possible phase diagram

- Seems exists a SL state at J2=0.1 with short spin-spin, bond-bond correlation length and a large spin gap.

- Even/odd topological sector splitting on even cylinders and dimerization effect on odd cylinders.

- Anisotropic effect on even cylinders YC4, YC8 and XC8, except YC6.

Rule out chiral order, calculate spin static structure to verify no other magnetic order exists, and determine the first phase transition point more accurately.