Is there subdiffusion in 2D?

Bar Lev and Reichman, EPL 113, 46001 (2016).
Transport in 2D
Numerical difficulties

Exact methods
• Exact diagonalization
  • Small size: 5x5
• tDMRG
  • Fast entanglement growth
  • Short times

Our method (perturbative)
• Kadanoff-Baym equations:
  \[ i\partial_t G^\xi (1, 1') = h (1) G^\xi (1, 1') + \hat{U}^{S_R} (1, \bar{1}) G^\xi (\bar{1}, 1') d\bar{1} + \hat{U}^{S^A} (1, \bar{1}) G^A (\bar{1}, 1') d\bar{1} \]

• Difficulties:
  • Resources: Memory: \( O(L^4 N_t^2) \)
    Complexity: \( O(L^6 N_t^3) \)
  • Symmetries must be conserved
  • Unstable (nonlinear equations)
• Distributed memory \( \rightarrow 1000 \) sites
Transport in 2D
Numerical difficulties

Our method (perturbative)

- **Kadanoff-Baym equations:**
  \[ i\hat{\partial}_t G^\xi (1,1') = h(1)G^\xi (1,1') \]
  \[ + \hat{U}^S R (1,\overline{1})G^\xi (\overline{1},1')d\overline{1} \]
  \[ + \hat{U}^S \xi (1,\overline{1})G^A (\overline{1},1')d\overline{1} \]

- **Difficulties:**
  - Resources: Memory: \( O(L^4 N_t^2) \)
    Complexity: \( O(L^6 N_t^3) \)
  - Symmetries must be conserved
  - Unstable (nonlinear equations)

- Distributed memory \( \rightarrow 1000 \) sites
Anderson-Hubbard Model in two dimensions

\[ \hat{H} = -\sum_{\langle ij \rangle \sigma} t_{ij} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + \sum_{i\sigma} h_{i\sigma} \hat{n}_{i\sigma} \]

- **Mean-square displacement**

\[ C(r,t) = \langle \delta \hat{n}_{r\sigma}(t) \delta \hat{n}_{0\sigma}(0) \rangle \]

\[ \langle r^2(t) \rangle = \frac{1}{V} \sum_r r^2 C(r,t) \]

\[ \alpha(t) \equiv \frac{d \ln r^2(t)}{d \ln t} \]

Bar Lev and Reichman, EPL 113, 46001 (2016).
Non-equilibrium perturbation theory

Dynamical exponent

\[ \alpha(t) = \frac{d \ln r^2(t)}{dt} \]

Possible anomalous thermalization!
Experiments
Mostly in cold atoms...

**Bloch group – 1d FH**


**Schneider group - 1d/2d FH**


**DeMarco group - 3d FH**


**Monroe group – Long range disordered Ising**


**Shahar group – a:InO**