The Gardner transition in glasses

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SIMONS FOUNDATION
The Gardner glass crackers

- A large research effort involving many PIs, affiliates, and collaborators.

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- Dedicated discussions during our retreat at Royaumont Abbaye (Oct. 2017), and Montpellier Gardner day (Jan. 2018).
Glass transition

- Fluid to amorphous glass at thermal equilibrium at finite $T$; occurs in all kinds of materials (liquids, polymers, colloids).

- Gardner transition occurs deep inside the glass phase.
Jamming transition

- **Athermal** packings of soft repulsive spheres (harmonic, WCA).

  ![Diagram of volume fraction and jamming transition](image)

  *Low $\varphi$: no overlap, fluid

  *Large $\varphi$: overlaps, solid

- Non-Brownian suspensions (below), hard grains (at), foams and emulsions of large droplets (above).

- A **Gardner phase** surrounds the jamming transition on both sides of $\varphi_J$. 
Glass & jamming transitions

- **Soft repulsive spheres** (harmonic, WCA) interpolate between many relevant glassy materials.

- **Jamming transition** deep inside glass phase, narrow critical region.

- **Gardner transition** may occur anywhere inside the glass phase; potentially relevant for all amorphous materials.
Why do we care about Gardner?

- A phase transition into a new amorphous phase of matter, which is marginally stable, as spin and electron glasses:
  - Density of states is changed (2015-18);
  - Elasticity is changed (2015-18);
  - Rheology is changed (2015-18);
  - Low-temperature transport properties are changed (2015-18).

- It unifies real space and mean-field descriptions of jamming and glasses, with the emergence of collective glassy excitations and soft modes.

- The critical exponents of jamming in dimensions $d \geq 2$ are quantitatively predicted from the marginal stability of the Gardner phase.

- 2015-18: We now understand how Gardner applies to glasses.
What is the Gardner transition?
Spin glass transition: Full-RSB

- Edwards-Anderson (1975), Sherrington-Kirkpatrick (1975) define the spin glass problem: $H = - \sum_{ij} J_{ij} S_i S_j + H \sum_i S_i$.

- de Almeida-Thouless (1978): mean-field spin glass transition in a field up to some critical field.

- Parisi (1979-83): In mean-field, spin glass transition described by continuous transition to replica symmetry broken spin glass phase, full-RSB.

- After 40 years of efforts, finite $d$ physics not completely understood.
  - $H = 0$ transition survives in $d = 3$, nature of low-$T$ phase?
  - $H > 0$ transition still controversial in $d < 4$.

- Gardner transition in same class as $H > 0$ transition. New motivations to revisit the problem; we benefit from 40 years of expertise.
Spin glass transition: 1-RSB

From 1-RSB to full-RSB

- In Potts glass, 1-RSB phase is unstable and undergoes a second continuous transition to a full-RSB phase.

- G. Parisi (2017): “The most impressive result was the discontinuous $q(x)$”.

**FIG. 1.** Schematic plots of the shape of the order function $q(x)$ in the two different PG phases. (a) The first PG phase, $T_2 < T < T_c$; (b) the second PG phase, $0 < T < T_2$. 
In J. Phys. A (1994), Parisi et al. write: “It is possible that [...] for lower values of $T$ a continuous symmetry breaking is needed to describe the system. This is what happens for the $p$-spin model [E. Gardner]. As we will discuss in the next sections this second transition would probably have no relevance from the physical point of view, since the system is not able to explore the lowest free-energy configurations."
On the nature of the low-temperature phase in discontinuous mean-field spin glasses

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Abstract. The low-temperature phase of discontinuous mean-field spin glasses is generally described by a one-step replica symmetry breaking (1RSB) ansatz. The Gardner transition, \textit{i.e.} a very-low-temperature phase transition to a full replica symmetry breaking (FRSB) phase, is often regarded as an inessential, and somehow exotic phenomenon. In this paper we show that the metastable states which are relevant for the out-of-equilibrium dynamics of such systems are \textit{always} in a FRSB phase. The only exceptions are (to the best of our knowledge) the \(p\)-spin spherical model and the random energy model (REM). We also discuss the consequences of our results for aging dynamics and for local search algorithms in hard combinatorial problems.

- State following construction to compute a (different) “Gardner transition” line.

- Physically relevant, as it adiabatically follows the evolution of a glass undergoing a second “glass transition”: the Gardner transition.
Back to glasses: RFOT theory

- Kirkpatrick-Thirumalai-Wolynes (1987-89) propose the 1-RSB transition of Potts and $p$-spin models as the correct mean-field description of the glass transition.

- From KTW ('87): “Another suggestive feature of the Potts glass is the appearance of lower-temperature transitions. This again often happens in structural glasses.” (With a reference to Johari-Goldstein, i.e. $\beta$-processes in supercooled liquids).

- KTW conjecture firmly confirmed by $d = \infty$ solution of glass transition (2013-15), which is at the root of this collaboration.

- Explicit $d = \infty$ results initially for the hard sphere potential.

- On increasing the packing fraction of the glass further, a 1-RSB to full-RSB Gardner transition is finally found, using state following.
Infinite dimension mean-field analysis.
Gardner transition in hard spheres

- Phase diagram obtained for hard spheres in $d = \infty$ (2014).

- The “stable glass” (1-RSB) is unstable towards the “marginal glass phase” (full-RSB). Key for jamming physics (critical exponents).
Gardner with soft interactions

- Phase diagram for soft spheres (WCA) in $d = \infty$ (2017).

- Gardner transition occurs as the glass is cooled further towards $T = 0$. Low-$T$ glass enters a marginal phase.

- Obvious new route to anomalous low-$T$ glass properties.
• Gardner transition is found for glasses prepared in many conditions. Suggests it is relevant for a broad class of materials.

• Warning from $d = \infty$: Not everywhere.
Let’s go down in dimensions:
1) RG, beyond mean-field theory.
Perturbative RG

- Field theory for de Almeida-Thouless transition. Expansion to one-loop order finds no fixed point for $d < 6$ (1980).


- A two-loop expansion (2017) finds a novel fixed point in the strong coupling regime, beyond the range of perturbative regime.

- Finite basin of attraction: not all microscopic models have a transition, but a transition can exist in $d < 6$. 
Real space RG

- Above 6 dimensions, Gaussian fixed point has a finite basin of attraction, as confirmed for Gardner transition itself (2015).

- Real space RG (2015) provides evidence for different non-perturbative, zero-temperature fixed points in large $d$. Activated dynamics?

*Migdal Kadanoff RG for $d > 8$.***
Non-Perturbative RG

- **Non-perturbative RG techniques** needed to treat the Gardner transition in finite $d$.

- **Encouraging NPRG results** for spin glass transition with $H = 0$ in $d < 6$ (2018), in progress for $H > 0$ (2015-18).
Let’s go down in dimensions:

2) Computer simulations and real space analysis.
A very large effort (2015-18): hard and soft potentials, in dimensions $d = 2, 3, 4$ to cover all possible cases (transition-no transition) and materials (from grains to colloids to dense liquids).
• Borrow from spin glass studies: (i) prepare two clones of the same glass, (ii) compare the distance between them ($\Delta_{AB}$) to thermal fluctuations ($\Delta$).

\[ \langle \Delta \rangle, \langle \Delta_{AB} \rangle \]

A: Close to jamming

B: Dense liquid

• In all cases, ergodicity is lost at $T < T_{\text{Gardner}}$. Two clones typically fall in different glass minima that become dynamically inaccessible.

• All this happens within a single glass configuration.
To age or not to age?

- Ergodicity breaking is very different in both cases.

- Emergence of collective aging behaviour near jamming. Average dynamics less interesting for dense liquids.

- The physics at play is likely non-universal.
The spin glass susceptibility \( \chi_{AB} = N[\langle \Delta_{AB}^2 \rangle - \langle \Delta_{AB} \rangle^2] \) reveals a phase transition.

Near jamming: It grows as \( T \rightarrow T_{\text{Gardner}} \), and increases logarithmically below; it remains featureless for dense liquids.

Simple models of dense liquids do not undergo a Gardner transition.
For most particles, $\langle \Delta_{AB} \rangle \gg \langle \Delta \rangle$. Visualisations and direct measurements reveal collective and spatially correlated behaviour.

All indications are consistent with an underlying phase transition, with physics reminiscent of the spin glass in a field.

Distinct from the jamming criticality itself: A true phase of matter.
For dense liquids, $\langle \Delta_{AB} \rangle$ dominated by small fraction (< 2%) of particles. For most particles, $\langle \Delta_{AB} \rangle \approx \langle \Delta \rangle$.

Visualisations reveals sparse, localised defects: glassy excitations that seem weakly coupled.
Localised defects in glasses

- Many energy minima available and visited at $T_{\text{glass}}$.

- Few minima visited as $T$ decreases, and nearby minima (two-level systems?) control the crossover at $T_{\text{Gardner}}$.

- Localised defects in glasses: dynamics, vibrational density of states, and rheology.

- Quasilocalised vibrational modes are being actively studied (2015-18).
Landscape near jamming

- Near jamming, vibrational soft modes are qualitatively different.

- We are exploring a hierarchical landscape near jamming, and the real space structure of glassy excitations in the whole Gardner phase.
Gardner transition in glasses

- A successful glass story, totally unexpected from the 1985 paper by Gross et al. which first mentions a Gardner transition.

- The collaboration is closing the Gardner problem in glasses:
  - detailed mean-field calculations;
  - new RG studies;
  - concerted numerical effort.

- We know when, where and why the Gardner transition exists, and in which glassy material a Gardner phase is physically relevant.

- Physical consequences of Gardner transition for dynamics (Reichman) and rheology (Wyart) are being actively explored.