Holographic Quantum Error Correction: What, How and Why?

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\[ e^{-iH_{\text{clever}}t_{\text{nice}}} \]

| \( \varphi \) | 0 | 0 }
It from Qubit: At a Glance

• Goal:
  – Advance our understanding of fundamental physics using information-theoretic methods

• 18 Principal Investigators
  – 6 countries
  – 5 continents
  – 2+ disciplines
An Active and Successful Collaboration

- 17 Conferences & Workshops
- 3 Schools
- 22 It from Qubit Fellows
- 255 articles
Highlights

- Quantum error correction & teleportation probe space-time behind horizons
- Holography with tractable boundary & bulk: Sachdev-Ye-Kitaev model
- Proof that black holes are nature’s fastest information scramblers: universal bound on many-body quantum chaos
- Information theoretic unification of renormalization monotones in quantum field theory
- Bulk space-time constraints from boundary information in AdS/CFT
- Complexity = Action conjecture: sharpened and tested
- Development of methods for simulating quantum field theory on quantum computers; small experimental demos
**Anti-de Sitter/Conformal Field Theory Correspondence**

**Conjecture:** Equivalence of string (gravity) theory in bulk with CFT on boundary [Maldacena’97]

**Question:** How are bulk degrees of freedom encoded in the boundary?
Relating bulk and boundary observables

Boundary in terms of bulk: *extrapolate*

\[ \mathcal{O}(\theta) = \lim_{r \to \infty} r^\Delta \phi(\theta, r) \]

Bulk in terms of boundary: *smearing*

\[ \phi(\theta, r) = \int K(\theta, r; \tilde{\theta}, \tilde{t}) \mathcal{O}(\tilde{\theta}, \tilde{t}) \, d\tilde{\theta}d\tilde{t} \]

\(K\) arises from solving some PDE’s (classical bulk field equations)

Don’t always need the whole boundary to reconstruct a given \(\varphi(\theta, r)\).

[Hamilton, Kabat, Lifschytz, Lowe 2006]
AdS/CFT as quantum error correction

Blue encloses “causal wedge” of boundary red

In empty AdS, for boundary interval, blue is a geodesic

All bulk operators in a boundary region’s causal wedge can be represented using only that boundary region.

A is sufficient to represent $\phi_2$

Write $A=A_1A_2$. Then $A_2A_3$ or $A_1A_3$ are each also sufficient.

Any two out of three of the boundary regions are sufficient.

[Hamilton, Kabat, Lifschytz, Lowe 2006][Almheiri, Dong, Harlow 2014]
Quantum error correction

Encoding a spin-1 into 3 spin-1 particles

$|\psi\rangle = \alpha |1\rangle + \beta |0\rangle + \gamma |0\rangle$

Quantum info $\varphi$ can be recovered from any two spins at time $t_{\text{nice}}$

$|\varphi\rangle = \alpha |1\rangle + \beta |0\rangle + \gamma |1\rangle$

$|\psi\rangle = \alpha (|1, -1, -1\rangle + |0, 0, 0\rangle + |1, 1, 1\rangle) + \beta (|1, 0, 1\rangle + |0, 1, -1\rangle + |1, -1, 0\rangle) + \gamma (|1, 1, 0\rangle + |0, -1, 1\rangle + |1, 0, -1\rangle)$
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Any two out of three of the boundary regions are sufficient!

Code: small perturbations of a fixed background state

[Hamilton, Kabat, Lifschytz, Lowe ‘94][Almheiri, Dong, Harlow 2014]
## Dictionary

### QEC language
- Physical qubits
- Code subspace
- Physical operators
- Logical operators
- Erasure recovery

- Many physical operators are same logical operator
- Physical operators depend on choice of code

### AdS/CFT
- Boundary
- Bulk fields in fixed bkgd
- Boundary operators
- Bulk operators
- Locality of boundary operator

- Ambiguity in bulk to boundary mapping
- State-dependent observables in AdS/CFT
AdS/QEC/CFT: The last three years

• Entanglement wedge reconstruction
  – Origin of bulk locality
  – Reconstruction of observables behind Rindler horizons

• Entropy as an observable

• Symmetry and QEC
  – No global symmetries in AdS/CFT
  – Exact versus approximate codes
The entanglement wedge

\[ S(A) = \frac{1}{4G_N} \min_{\gamma_A} \text{area}(\gamma_A) \]

Causal wedge: LEFT + RIGHT

Entanglement wedge: LEFT + MIDDLE + RIGHT

Hamilton, Kabat, Lifschytz, Lowe:
Decode all bulk operators in the causal wedge

Information theoretic methods:
Decode all bulk operators in entanglement wedge

\[ \mathcal{O}_A = \frac{d}{d\epsilon |_{\epsilon=0}} H_A [\rho_{bulk} + \epsilon \mathcal{O}_{bulk}] \]

[Czech, Karczmarek, Nogueira, van Raamsdonk ’12][Headrick, Hubeny, Lawrence, Rangamani ’14][Pastawski, Yoshida, Harlow, Preskill ’15][Hayden, Nezami, Qi, Thomas, Walter, Swingle ’16][Jafferis, Lewkowycz, Maldacena, Suh ’16][Dong, Harlow, Wall ’16][Faulkner, Lewkowycz ’17][Cotler, Hayden, Salton, Swingle, Walter ’17]
Bulk Locality from Error Correction

Two observables that are spacelike in the bulk should commute. Do they?

Can find boundary representations acting on disjoint sections of boundary. YES!
Complementary recovery

Causal wedge: LEFT + RIGHT

Entanglement wedge: LEFT + MIDDLE + RIGHT
Bulk reconstruction behind Rindler horizons

Quantum error correction allows for reconstruction of observables that are not in causal contact with the given boundary region.

Toy version of reconstructing observables behind black hole horizons.
Entropy as an Observable

Ryu-Takayanagi formula (to leading order in $1/G_N$):

$$S(A) = \frac{1}{4G_N} \min_{\gamma_A} \text{area}(\gamma_A)$$

Nonlinear functional of state

Observable
Entropy as an Observable

Ryu-Takayanagi formula (to leading order in $1/G_N$):

$$S(A) = \frac{1}{4G_N} \min_{\gamma_A} \text{area}(\gamma_A) \quad \text{to leading order in } 1/G_N$$

$$S(A) = -\text{tr} \rho \log \rho$$

Observables

Nonlinear functional of state

Each region $A_1$, $A_2$, $A_3$ participates in code but individually must not reveal any information about $\phi$

Entanglement between $A_i$ & $A_i^c$ contains a portion which insensitive to state

$$S(A) = \text{tr} \mathcal{L} \rho + S_{\text{fields}}$$

[Harlow '17]
No global symmetries in AdS/CFT

- Global symmetry:
  - Maps local operators to local operators
  - Acts faithfully on local operators
  - Commutes with energy-momentum tensor
  - \( U(g) = \prod_i U(g, R_i) \)

If the AdS quantum gravity theory has a global symmetry, there must be a local bulk operator transforming faithfully to a different operator localized to the same region.

Contradiction!

[Harlow, Ooguri ‘18]
Covariant Quantum Error Correction

Code subspace $S$ of $\mathcal{H} = \bigotimes_{i=1}^{n} \mathcal{H}_i$  \[ E : S \leftrightarrow \mathcal{H} \]

Can such codes exist?  Generators: $T_s \ E = (T_1 + T_2 + \ldots + T_k) \ E$. Charge is conserved.

But error correcting property requires $\langle T_i \rangle = \text{const}$ so $T_s$ is trivial!

Loophole: small errors in large codes

[Hayden, Nezami, Popescu, Salton ’16][Faist, Nezami, Albert, Salton, Hayden, Preskill ’18]
Covariant Quantum Error Correction

Code subspace $S$ of $\mathcal{H} = \bigotimes_{i=1}^{n} \mathcal{H}_i$  

$E : S \rightarrow \mathcal{H}$

- Driving through the loophole
  - Time evolution in AdS/CFT (generalize to overlapping generators)
  - Error correction of quantum reference frame information
  - Universal fault-tolerant transversal gates for quantum computation?

[Hayden, Nezami, Popescu, Salton ‘16][Faist, Nezami, Albert, Salton, Hayden, Preskill ‘18]
Future directions: incremental

• Explicit calculations
  – Bulk reconstruction beyond the causal wedge

• Beyond complementary recovery
  – Structural results depend on operator algebra quantum error correction
  – Breaks down in presence of large amount of bulk entropy

• Systematically handle vanishing but nonzero error in reconstruction
Future directions: big leap

• Quantum error correction has proven a powerful conceptual framework for understanding AdS/CFT
• We don’t live in AdS
• First step: Asymptotically flat space
  – BFSS matrix model
    • Search for analog of Ryu-Takayanagi
    • Identify protected degrees of freedom: scrambling
• Bigger step: de Sitter
  – Possible approach:
    • Dong, Silverstein & Torroba: Proposed extension of Ryu-Takayanagi
Von Neumann entropy in AdS/CFT

Ryu-Takayanagi proposal for bulk formula:

\[ S(A) = \frac{1}{4G_N} \min_{\gamma_A} \text{area}(\gamma_A) \]

Minimize over spatial bulk surfaces \( \gamma_A \) homologous to \( A \).

Generalizes black hole entropy to wide class of spatial regions!
Spatial slice of anti-de Sitter space

- Hyperbolic space
- Fish-counting metric
- Geodesics (straight lines) follow fish
- Can place matter deep inside AdS