Wormholes and Entanglement

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Outline

• Black holes as quantum systems
• Schwarzschild wormhole and entangled states.
• Traversable wormholes.
• Simple dynamics in Nearly-AdS$_2$
• The Hayden Preskill problem revisited.
• Traversable wormhole solution in 4d.
Black holes as quantum systems

• A black hole seen from the outside can be described as a quantum system with $S$ degrees of freedom (qubits).
Simplest solution

Vacuum solution.
Two exteriors, sharing the interior.

Schwarzschild, Eddington, Lemaitre, Einstein, Rosen, Finkelstein, Kruskal

Singularity

Left exterior

Right exterior
Wormhole interpretation.

There are two asymptotic regions. The blue spatial slice contains the EinsteintRosen bridge connecting the two regions, not in causal contact and information cannot be transmitted across the bridge. This can easily be seen from the Penrose diagrams and is consistent with the fact that entanglement does not imply nontlocal signal propagation.

Another representation of the blue spatial slice of figure y contains a neck connecting two asymptotically flat regions. Here we have two distant entangled black holes in the same space. The horizons are identified as indicated. This is not an exact solution of the equations but an approximate solution where we can ignore the small force between the black holes.

All of this is well known, but what may be less familiar is a third interpretation of the eternal Schwarzschild black hole. Instead of black holes on two disconnected sheets, we can consider two very distant black holes in the same space. If the black holes were not entangled, we would not connect them by an EinsteintRosen bridge. But if they are somehow created at $t = \text{entangled state}$, the bridge represents the entanglement. Of course, in this case, the dynamical decoupling is not appropriate.

Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar
Non-traversable due to:

1) Einstein’s equations

2) Average null energy condition

\[ \int dx^+ T_{++} \geq 0 \]
Wormhole and entangled states

There are two asymptotic regions. The blue spatial slice contains the Einstein-Rosen bridge connecting the two regions not in causal contact and information cannot be transmitted across the bridge. This can easily be seen from the Penrose diagrams and is consistent with the fact that entanglement does not imply nonlocal signal propagation.

(a)
(b)

Figure z: Another representation of the blue spatial slice of figure y. It contains a neck connecting two asymptotically flat regions. Here we have two distant entangled black holes in the same space. The horizons are identified as indicated. This is not an exact solution of the equations but an approximate solution where we can ignore the small force between the black holes.

All of this is well known, but what may be less familiar is a third interpretation of the eternal Schwarzschild black hole. Instead of black holes on two disconnected sheets, we can consider two very distant black holes in the same space. If the black holes were not entangled we would not connect them by an Einstein-Rosen bridge. But if they are somehow created at $t = \text{int} \text{ernal}$, the bridge represents the entanglement. See figure zobp. Of course, in this case, the dynamical decoupling is not connected through the interior.

In a particular entangled state

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

W. Israel, JM ER = EPR, JM. Susskind
• In this set up the horizon is “exact”, it does not come from coarse graining.
There are two asymptotic regions. The blue spatial slice contains the Einstein-Rosen bridge connecting the two regions, which are not in causal contact and information cannot be transmitted across the bridge. This can easily be seen from the Penrose diagrams and is consistent with the fact that entanglement does not imply nonlocal signal propagation.

(a) Another representation of the blue spatial slice of figure y. It contains a neck connecting two asymptotically flat regions. Here we have two distant entangled black holes in the same space. The horizons are identified as indicated. This is not an exact solution of the equations but an approximate solution where we can ignore the small force between the black holes.

All of this is well known, but what may be less familiar is a third interpretation of the eternal Schwarzschild black hole. Instead of black holes on two disconnected sheets, we can consider two very distant black holes in the same space. If the black holes were not entangled, we would not connect them by an Einstein-Rosen bridge. But if they are somehow created at $t = 0$ with entangled states, the bridge between them represents the entanglement. See figure z. Of course, in this case, the dynamical decoupling is not valid. The signal is just behind the horizon, but cannot be extracted by the right observer.
“What happens in the interior stays in the interior”
Including gravitational dynamics

.. in a simple case
The surprisingly simple gravitational dynamics of N-AdS$_2$

NAdS$_2$ = AdS$_2$ + location of boundary

Dynamics of the boundary is SL(2) invariant.

Proper time along the boundary = time of the dual quantum system. Motion of the boundary = relation between the two times.

\[
(H_{L_{Bdy}} \times H_{bulk} \times H_{R_{Bdy}})/SL(2, R)
\]

Kitaev Suh
JM Stanford Yang
...

The boundary trajectory gets a “kick” determined by local energy momentum conservation. You can see less of the inside.

Dynamics

Emission of a bulk excitation

New position of the horizon
Could you see more?

- Yes!
- Introducing an interaction between the two sides.

Gao Jafferis Wall

JM, Stanford, Yang Susskind, Yhao

...
Interaction between the two boundaries

\[ \phi \text{ is a bulk field} \]

Insert this in the path integral

\[ e^{ig\phi_L(t_L)\phi_R(t_R)} \]

approximate

\[ e^{ig\langle \phi_L(t_L)\phi_R(t_R) \rangle} \]

Force between the two boundaries.
(Can be attractive for the right sign of g).
Kicks the trajectories inwards.
Creates negative null energy.

More precisely: we have N bulk fields to enhance the effect.
Does this interaction violate causality?

• If this is the bulk dual of two separate quantum mechanical systems, then we are free to introduce an interaction between the two quantum mechanical systems.

• We could have two entangled black holes that are close to each other at some instant.

• The full causality structure depends on how this geometry is embedded in a bigger space.

• We still need some sense of time ordering.
Quantum Teleportation interpretation

Measure
\[ \phi_L \rightarrow \sigma_L \]

Act on the right with
\[ e^{i g \sigma_L \phi_R(t_R)} \]

From the point of view of the right we get the same, whether we measure or not.

Bennett, Brassard, Crepeau, Jozsa, Peres, Woetters
• The interesting fact is how the teleportee feels.
• It feels that it went through empty space.
• It offers an opportunity to explore a bigger region of AdS$_2$ than the Rindler wedge. Or to explore the “interior”.
What if we want to send too much information?

The insertion of the message also gives a small kick to the trajectory.

Moves the insertion points of the non-local operator away from each other

\[ \langle \phi_L(t_L)\phi_R(t_R) \rangle \] becomes smaller

Attractive force weakens → no opening of the wormhole.
Comments on teleportation

- The message is “secret” because it goes through a wormhole!
- Where is the message after you do the measurement on the left, but before you extract it?
Message is lost to the left after the measurement

It is also not accessible to the right if the right observer does not know the result of the measurement!
Message is lost to the left after the measurement

It is accessible to the right after she learns the result of the measurement.

New knowledge, if used wisely, can expand your horizons!
Geometry seems to encode these properties “better” than the standard QM description!
Application to the Hayden Preskill problem
Figure from the paper of Preskill and Hayden.

Duplication of information in the geometry. But it cannot be checked.
Bob’s computer

Old black hole

Maximally entangled
Bob produces a second black hole, maximally entangled with the first. (This is hard to do)

Harlow Hayden

Old black hole

Maximally entangled
Bob produces a second black hole, maximally entangled with the first.
Say they are nearly AdS$_2$ black holes...
Bob’s black Hole, which is Part of Bob’s computer

Alice’s black hole

Alice’s message

Trajectories of the boundaries Before Bob catches the Hawking mode
Bob gets some radiation and feeds it to his computer.

Bob’s black Hole, which is Part of Bob’s computer

Feeding his computer = black hole.

Bob getting radiation

Trajectories of the boundaries before Bob catches the Hawking mode

Trajectories of the boundaries after Bob catches the Hawking mode

Alice’s black hole

Alice’s message
Bob now gets the message at P

Bob’s black Hole, which is Part of Bob’s computer

Alice’s black hole

Alice’s message

Trajectories of the boundaries
Before Bob catches the Hawking mode
The message switched sides!

Bob’s black Hole, which is Part of Bob’s computer

Feeding his computer = black hole.

Backward extrapolation Of the state of Alice’s boundary after Bob’s extraction, using the unperturbed Hamiltonian.

Trajectories of the boundaries Before Bob catches the Hawking mode

New horizon

Bob getting radiation
Before **transfer**: Alice has the message but Bob does not.

After **transfer**: Bob has it but Alice does not!

**Bob getting radiation**

Backward extrapolation
Of the state of Alice’s boundary after Bob’s extraction, using the unperturbed Hamiltonian.

**Only one copy of the message throughout!**
More like the HP figure

Now Bob cannot get the message!, it is still in Alice’s possession.
Now Alice send is later...

Bob will not get it here.

Bob gets the machinery that sent Alice’s message, but with no message.
Now Alice send is later...

Bob will not get it here.

Bob can extract that machinery

Bob getting radiation

Alice’s message

Alice’s black hole
Now Alice send is later...

Bob will not get it here.

Bob can extract that machinery

Alice’s message

Bob getting radiation

Alice’s black hole
Now Alice send is later...

Bob can evolve his side backwards in time and recover the message here.
Summary

• The process of extracting the message puts it out of reach from Alice.
• The message is never duplicated in the bulk picture.
• No need to invoke unknown new transplanckian physics to solve the no-cloning problem.
• All understandable from standard rules of gravity on the wormhole geometry.
• Assumes ER=EPR.
• What is the geometric picture of the process leading up to the two entangled black holes?
When doing the complicated quantum computation that extracts the message,

It is important to include the spacetime generated by the quantum computation

Like the Maxwell demon:
The quantum computer also has a gravity dual, which is connected to the interior of the original black hole
We gave a geometric interpretation of when the decoding is done in a particular way.

What if it is done in different ways? Do they all have a geometric interpretation?
How “realistic” is the TFD state?

• Is this state simple to make?
• Isn’t it hopelessly fine tuned and unstable?
• Is is just an unphysical mathematical idealization?

• All of these were said of the single sided Schwarzschild solution...
Methods to prepare the TFD

Cottrell, Freivogel, Hofman, Lokhande
Martyn, Swingle
Wu, Hsieh

JM Qi
TDF for NAdS$_2$ as the ground state of a coupled system

• Keep the interaction for ever.

\[ S_{int} = \mu \int du \phi_L(u) \phi_R(u) \]

• Get an “eternally” traversable wormhole \(\rightarrow\) similar to the whole global AdS$_2$ spacetime.
NAdS$_2$ gravity + Interaction

\[ H_L + H_R + H_{int} \]

Boundaries now move “straight up”

Signals can now propagate from one boundary to the other.
“Practical” method to make the TFD double

• Couple the two systems.
• Couple them weakly to a heat sink.
• Wait.
• Get to the ground state.

\[ |G\rangle \sim |TFD\rangle \]

• If we turn off the interaction, it will evolve as before.
Turn off the interaction at \( t=0 \)

This part of the diagram represents the evolution of the TFD state with the decoupled Hamiltonian.

The full spacetime diagram represents the full evolution of the state.
Interactions behind the horizon
\[ H = H_L + H_R \]

Evolve the TFD, backwards, insert some excitations with unitary operators.
We expect that the initial state is describing the Wheeler de Wit patch
We can evolve it with the decoupled hamiltonian

\[ H = H_L + H_R \]

From the boundary theory they should NOT Interact in any way.

Fortunately, their interaction is behind the horizon so we cannot see it from either boundary.
We can evolve it with the coupled Hamiltonian

$$H = H_L + H_R + H_{int}$$

We can now see the interaction.

It is OK because the underlying Hamiltonian has an interaction between the two sides.

Make that interaction behind the horizon more real. But always through the lens of a particular evolution.

Acting with two sided operators we can see the interior
• The above discussion used gravity.
• Can the same process happen in a simple quantum mechanical model?
• The SYK model!
What is the precise relation between SYK and AdS$_2$?
SYK model

Low energies

Conformal invariant part + reparametrizations

Not the same

QFT on AdS$_2$ + boundary dynamics

same

Schwarzian action

Boundary gravitons

$S = -C \int du \{ f(u), u \}$

Emergent reparametrization symmetry which is spontaneously and explicitly broken

- Low temperature entropy
- Gravitational backreaction
- Chaos exponent
- Wormhole traversability (location of horizon)

Kitaev Suh JM, Stanford Z. Yang
What do these two dimensional black holes and wormholes have to do with our real four dimensional world?
Near extremal black holes

\[ M \geq Q \]
\[ M \sim Q \]

N-AdS$_2 \times S^2$

horizon
Interaction between two black holes?
→ Get them relatively close together
Analogy: Van der Waals interaction

Two neutral atoms exchanging photons.

\[ H_{int} \propto \frac{\vec{d}_L \cdot \vec{d}_R}{d^3} \]

\( d \) small enough so that \( 1/d \) is larger than the gap between the ground state and the next states.

Entangle the two atoms.
Here we get the following traversable wormhole geometry

Figure: Wheeler 1966

JM, Milekhin, Popov
The theory

\[ S = \int d^4x \left[ R - F^2 + i\bar{\psi} \slashed{D}\psi \right] \]

Einstein + U(1) gauge field + massless charged fermion

Could be the Standard Model at very small distances, with the fermions effectively massless. The U(1) is the hypercharge. SU(3) x SU(2) x U(1).

\[ l_{\text{Planck}} = 1 \]
Black holes with opposite magnetic charge
4 d massless fermion $\rightarrow$
set of two dimensional massless fermions along the field lines

Similar to:
Fermion trajectories

Charged fermion moves along this closed circle.
Assume: “Length of the throat” is larger than the distance.

\[ L \gg L_{\text{out}} > d \]

Casimir energy is of the order of

\[ E \propto - \frac{q}{L} \]

Full energy also need to take into account the conformal anomaly because AdS$_2$ has a warp factor. That just changes the numerical factor.
• Including this energy in Einstein’s equation we find a solution with the wormhole shape.

• Has no horizon.

• Represents a pair of entangled black holes in a state similar to the TFD.
• To prevent the two mouths from falling into each other → make them orbit around each other.

• Gives a long lived state.
Question

• How quickly does the wormhole if we started from two separate black holes?

• In a similar SYK model? $\Rightarrow$ it seems that it happens in a time of order $N^0$.

• In 4 dimensions $\Rightarrow$ We don’t know.
Entropy and entanglement

• Total spacetime has no entropy and no horizon.

• If we only look at one object → entanglement entropy = extremal black hole entropy

• Wormhole = two entangled black holes

• Total Hamiltonian

\[ H = H_L + H_R + H_{\text{int}} \]

Generated by fermions in exterior
What is a black hole?

• State?
• State and some particular evolution law, which does permit us to access some region.

• Same state can be evolved in two ways:
  – Decoupled evolution \(\rightarrow\) two black holes
  – Coupled evolution \(\rightarrow\) traversable wormhole, no horizon.
Conclusions

• The traversable wormhole construction lets us explore the connected spacetime produced by an entangled state: the TFD.
• New light on the Hayden Preskill process.
• Inspiration to build traversable wormhole solutions in four dimensions.
Length $L$ as $d$ increases

$$E_c \propto \frac{1}{4 \frac{1}{L}} - \frac{1}{L + d}$$

Conformal anomaly  
Casimir cylinder

Stops being classical