Scrambling and (Quantum) Spacetime

Brian Swingle (UMD & IAS)

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Overview

• Approaching scrambling via chaos and the quantum butterfly effect, connections to information flow and spacetime geometry
• Universal characteristics of scrambling in chaotic, locally interacting quantum systems (finite N)
• Frontiers
  • Quantum Lyapunov spectrum
  • Scrambling experiments and simulating quantum gravity
  • Universality from quantum spacetime superpositions?
The Butterfly Effect.

flap flap

Yesss... Yesss...

by J.L. Westover

www.mrlovenstein.com
Quantum butterfly effect

\[ W(t) = e^{iHt} \cdot W \cdot e^{-iHt} \]

backward perturbation forward
Spacetime diagram of qubits with local interactions
Fixed time snapshots of qubits with all-to-all interactions
Support of $W(t)$ grows exponentially with time ...
Quantum: probability amplitudes for all possible ways to spread

\[ \sigma_1^\alpha \otimes I_2 \text{ or } I_1 \otimes \sigma_2^\alpha \text{ or } \sigma_1^\alpha \otimes \sigma_2^\beta \]

\[ \sigma_1^x \otimes I_2 \]
We must specify more than just where the operator is \( \rightarrow \)
Heisenberg operator has a “quantum shape” (wavefunction)

We can try to feel this “quantum shape” by probing \( W(t) \) in different states of the system (typically, thermal-like states)
Commutator

• Expand operator in a complete basis:

\[ W(t) = \sum_{\alpha_1 \cdots \alpha_n} a(\alpha_1 \cdots \alpha_n) \sigma^{\alpha_1} \otimes \cdots \otimes \sigma^{\alpha_n} \]

• Study dynamics in a highly excited state

\[ G_R = -i \langle \psi | [W(t), V] | \psi \rangle \]

\[ G_R \rightarrow -i \text{Tr}(\rho_T [W(t), V]) = 0 \]
Commutator$^2$

- Can better probe the growth of $W(t)$ by studying the squared commutator:

$$C(t) = \langle [W(t), V]^\dagger [W(t), V] \rangle_T$$

- For simplicity, assume $W$ and $V$ are unitary

$$C = 2 - 2 \text{Re}[F]$$

$$F(t) = \langle W(t)^\dagger V^\dagger W(t)V \rangle_T$$

Out-of-time-order correlator (OTOC)
Local vs all-to-all at early time \((C \ll 1)\)

- Local: \(W\) and \(V\) separated by \(x\)
  \[ C \sim e^{\lambda(t-x/v_B)} + \cdots \]

- All-to-all:
  \[ C \sim \frac{1}{N_{\text{dof}}} e^{\lambda t} + \cdots \]

- AdS/CFT has both structures:
  \[ C \sim \frac{1}{N_{\text{dof}}} e^{\lambda(t-x/v_B)} + \cdots \]

actually, not the most general form .... (see later)
Commutators and quantum chaos

• Semiclassical limit

\[ V = e^{i\frac{q}{a}}, \quad W = e^{i\frac{p}{b}} \]

\[ F(t) \approx e^{-\langle [q_t, p] \rangle / ab} + \ldots \]

\[ \langle [q_t, p] \rangle \approx i\hbar \{q_t, p\}_{PB} = i\hbar \left( \frac{\partial q_t}{\partial q} \right) = i\hbar e^{\lambda_L t} \]

• Chaos bound

\[ \tilde{F} = 1 - \epsilon e^{\lambda_L t} \]

\[ \lambda_L \leq 2\pi T \]

“thermally regulated” OTOC, general statement is about logarithmic derivative

[Larkin-Ovchinnikov 1968]

[Maldacena-Shenker-Stanford ‘15]
Chaos bound and black holes

• Black hole scrambling

\[ F_{\text{Einstein}} \sim 1 - \frac{e^{2\pi T t}}{S} \]

early time, saturation later

[Shenker-Stanford ’13, Kitaev ‘14]

• Calculate by studying high energy collisions near a black hole horizon

• Within AdS/CFT, scrambling has a precise information theoretic meaning and probes the near horizon geometry of black holes
Information velocity $v_I$

If $v > v_I$, then entanglement can be recovered from $A_t^{(v)}$

If $v < v_I$, then entanglement can be recovered from $[A_t^{(v)}]'$
Butterfly speed = information speed

*at least, for chaotic translation invariant systems

• Upper bound: the butterfly speed represents a maximum speed for excitations about a given state, so information can’t move faster than the butterfly speed

\[ v_B \geq v_I \]

• Lower bound: the butterfly speed measures the size of the region over which information is scrambled, the rest of the chain functions like the early radiation in Hayden-Preskill, the transfer of any piece of butterfly region to the complement would allow the information to be recovered in the augmented complement

\[ v_B \leq v_I \]

[conjectured extension of Hosur-Roberts-Qi-Yoshida, Kitaev-Yoshida]
Rule (follows from RT+FLM): If the bulk particle is in entanglement wedge of a boundary region $A$, then $A$ is maximally entangled with $R$. 

$A^c$ $A$ $\gamma_A$ $\sigma_A$ $R$ $A^c$
Information velocity

\[ v_B = \sqrt{\frac{d+z-\theta}{2(d-\theta)}} \left( \frac{T}{T_0} \right)^{1-1/z} \]

Boundary (CFT)

Black hole horizon

[Stanford-Mezei ‘16, S: https://sites.google.com/site/physicsmonkey/qscrambling]
Emergent speed limits: scrambling also helps us understand the emergence of spacetime causality
Measuring scrambling

\[ F = \langle \psi | W_t^\dagger V^\dagger \rangle \left[ W_t V |\psi\rangle \right] \]

[S-Bentsen-Schleier Smith-Hayden '16]
Additional protocols

• “Quantum clock” [Zhu-Hafezi-Grover ‘16]
  • Control qubit also controls direction of time; reduces some sources of error

• Purity-like measurement [Yao-Grusdt-S-Lukin-StamperKurn-Moore-Demler ‘16]
  • No time-reversal required, but must measure many-body overlap
  \[ \tilde{F} = \text{tr}(\rho^{1/2}W(t)V^{\dagger}\rho^{1/2}W(t)V) \]

• Weak measurement [Yunger Halpern ‘16, Yunger Halpern-S-Dressel ‘17]
  • Coherent ancilla not needed

• Decoding approach [Yoshida-Kitaev ‘17]

• Non-eq thermo [Campisi-Goold ‘16]
Experiments so far (I)

• OTOCs in trapped ion quantum simulator

Quantum Physics
Measuring out-of-time-order correlations and multiple quantum spectra in a trapped ion quantum magnet
Martin Gärttner, Justin G. Bohnet, Arghavan Safavi-Naini, Michael L. Wall, John J. Bollinger, Ana Maria Rey
(Submitted on 31 Aug 2016)

Appeared in Nature Physics

~ 100 spins
Hilbert space size: ~ 100

\[ H = \chi S_z^2 \]

• OTOCs in NMR quantum simulator

ArXiv.org > cond-mat > arXiv:1609.01246
Condensed Matter > Strongly Correlated Electrons
Measuring out-of-time-order correlators on a nuclear magnetic resonance quantum simulator
Jun Li, Ruihua Fan, Hengyan Wang, Bingtian Ye, Bei Zeng, Hui Zhai, Xinhua Peng, Jianguo Du
(Submitted on 5 Sep 2016 (v1). Last revised 24 Sep 2016 (this version, v2))

Appeared in PRX
Experiments so far (II)

• OTOCs in spin chains with NMR

\[ H = u \sum_{j=1}^{L-1} \frac{J}{2} (\sigma^j_x \sigma^{j+1}_x - \sigma^j_y \sigma^{j+1}_y) + b \sum_{j=1}^{L} \sigma^j_z \]

\[ + g \sum_{j=1}^{L} h_j \sigma^j_z + v \sum_{j=1}^{L-1} \frac{J}{2} (\sigma^j_x \sigma^{j+1}_x + \sigma^j_y \sigma^{j+1}_y - 2 \sigma^j_z \sigma^{j+1}_z) \]

• OTOCs with BEC

\[ H(t) = p S_z + \chi S_z^2 \sum_k \delta(t - k\tau) \]
Universal commutator dynamics
Two classes (at least)

1. Large N models (AdS/CFT, SYK, O(N))

\[ C \sim \frac{1}{N_{dof}} e^{\lambda (t-x/v_B)} + \ldots \]


2. Random unitary circuit model

\[ C \sim \text{erf} \left( \frac{x-v_B t}{\sqrt{D t}} \right) \]

[Nahum/Haah/Vijay, Keyserlingk/Rakovszky/Pollman/Sondhi, see also: follow up work with symmetries Khemani et al.]
General early growth form

\[ C(r, t) \sim \exp \left( -\lambda \frac{(r - v_B t)^{1+p}}{v_B (v_B t)^p} \right) \]

- \( p = 0 \): Semi-classical/Large N/holographic duality
  - Sharp wavefront
- \( p = 1 \): Random circuit models
  - Diffusively broadened wavefront
- Other models fit into this framework, e.g. non-interacting

[Xu/S arXiv:1802.00801, see also: Khemani/Huse/Nahum 1803.05902]
Model: Brownian coupled clusters

\[ O(t) = \sum a(\sigma_1, \sigma_2, ..., \sigma_L) \sigma_1 \sigma_2 ... \sigma_L \]

\[ \tilde{h}(w) = D(w) \left| a \right|^2 \quad C(r, t) \sim \frac{w(r, t)}{N} \]

\[ \partial_t \tilde{h} = \sum_r \left[ \left( -\gamma_r^+(w) \tilde{h}(w) + \gamma_r^+(w + 1) \tilde{h}(w + e_r) \right) + \left( -\gamma_r^-(w) \tilde{h}(w) + \gamma_r^-(w - 1) \tilde{h}(w - e_r) \right) \right] \]

\[ + \sum_{\langle rr' \rangle} \left[ \left( -\gamma_b^+(w, w_{r'}, w_r) \tilde{h}(w) + \gamma_b^+(w_r + 1, w_{r'}) \tilde{h}(w + e_r) \right) \right. \]

\[ + \left[ \left( -\gamma_b^-(w, w_{r'}, w_r) \tilde{h}(w) + \gamma_b^-(w_r - 1, w_{r'}) \tilde{h}(w - e_r) \right) \right] \]

\[ + [r \leftrightarrow r'] \]

[Xu-S 1805.05376, related results for single SYK cluster: Roberts-Streicher-Stanford]
• Infinite on-site degrees of freedom (infinite N)

Fisher-Kolmogorov-Petrovsky-Piskunov (FKPP) type equation

\[ \partial_t C = (2 - C) \left( \frac{\nu_B^2}{\lambda_L} \partial_r^2 C + \lambda_L C \right) \]

Growth + Saturation

\[ C(r, t) \sim e^{\lambda_L (t - \frac{r}{\nu_B})} \quad p = 0 \]

[FKPP (without noise) in other models: Faro/Ioffe/Aleiner, Chowdhury/S, Aavishkar/Chowdhury/S/Sachdev, ...]
The large $N$ expansion

FKPP Equation with noise induced by quantum fluctuations

$$\partial_t C = (2 - C)(\partial_r^2 C + C) + f_{noise}$$

$$f_{noise} \sim \sqrt{\frac{C}{N}} \eta(r, t) \rightarrow D \sim \frac{1}{\log^3 N}$$

Growth + diffusion + saturation

$$p = 0 \rightarrow p = 1$$

Spin chains: low entanglement outside front

\[ H = -J \sum_{r} \sigma_{r}^{z} \sigma_{r+1}^{z} - h_{x} \sum_{r} \sigma_{r}^{x} - h_{z} \sum_{r} \sigma_{r}^{z} \]

\[ W(t) \approx \]

[Xu/S arXiv:1802.00801, see also: Bohrdt et al 1612.02434]
Constant C contours

\[ C(x, t) = \exp \left( -\lambda \frac{(x - vt)^{1+p}}{v(vt)^p} \right) \]

\[ x(t) = vt + \left( \frac{v}{\lambda} (vt)^{p \log C} \right)^{\frac{1}{1+p}} \]

[Xu-S '18]
Frontiers
Quantum Lyapunov spectrum

• Is there a notion of a spectrum of chaos exponents? Relation to entanglement growth or some quantum generalization of KS entropy?

• Previous work includes study of commutator spectrum in single-body problem [Galitski et al.], mapping to effective non-linear model [Green et al.]; also many attempts to construct KS entropy

• A proposal [Gharibyan-Hanada-Tezuka-S]
  • Construct a matrix of “phase space derivatives”
  • Resulting eigenvalues generalize chaos exponent
  • We find they obey random matrix statistics

\[ M_{ij} = \{ \chi_i(t), \chi_j(0) \} \]
\[ L_{ij} = \langle M_{ik}^\dagger M_{jk} \rangle \]
Making wormholes

w/ John Martyn 1812.01015
see also Cottrell at el. 1811.11528
and Wu et al. 1811.11756

FIG. 9: Exact and PSA (using exact unitary) TFD correlation functions $\langle \chi_\alpha(t) \otimes \chi^*_\alpha \rangle$ for the SYK model with $N = 20$ Majorana fermions per subsystem and $T = 0.2$. $\alpha \in \{4, 9, 17, 20\}$ are displayed as representatives of all correlation functions.
Scrambling universality in gravity?

• From CFT point of view, the basic mechanism (superpositions of operator strings) operates (maybe suppressed by strong coupling?)

• Chaos amplifies small fluctuations, including quantum fluctuations; moreover, in a quantum system, amplification is accompanied by noise production (theory of quantum amplifiers)

• Somewhat analogous to the stretching of quantum fluctuations by inflation, what if chaos amplifies quantum fluctuations to macroscopic scale leading to a quantum superposition of spacetimes

\[ |\Psi\rangle = \sum_h \sqrt{P[h]} |h\rangle \]
Summary

• Scrambling is a rapidly developing field both in theory and experiment; connections to quantum chaos, speed limits, electrical and thermal transport, wormholes and quantum teleportation, ...

• Progress so far includes a refined physical picture, a broad calculational framework, well-supported conjectures for universal physics, and first experimental steps

• Frontiers include relations to other measures of quantum chaos, exponential growth well ahead of chaos front?, simulating quantum gravity in experiment, quantum spacetime and chaos, ...
Bonus: (in progress) locality + fast scrambling?