Gravitational Physics from Quantum Information Constraints

Mark Van Raamsdonk

IFQI's: Don Marolf Rob Myers Horacio Casini Juan Maldacena Brian Swingle Tadashi Takayanagi

IFQ P.D.s: Felix Haehl Eliot Hijano Onkar Parrikar Charles Rabideau Aitor Lewkowycz Nima Lashkari

† many students + other collaborators
Setting: the AdS/CFT correspondence

\[ i\hbar \frac{d\vert \Psi \rangle}{dt} = H \vert \Psi \rangle \]

Non-gravitational quantum system (often: CFT)

Higher-dimensional (quantum) gravitational system

encoding
Setting: the AdS/CFT correspondence

\[ i\hbar \frac{d\left| \Psi \right\rangle}{dt} = \mathcal{H} \left| \Psi \right\rangle \]

Non-gravitational quantum system
(often: CFT)

Quantum information ideas are fundamental to how this works.

Higher-dimensional (quantum) gravitational system
1970's observations: Black hole horizon areas behave like entropy

Bekenstein, Hawking, ...

\[ \Delta A > 0 \]

\[ dM = \frac{\kappa}{8\pi} dA \]
$T > \frac{1}{K}$

Thermal state of CFT

Black hole in AdS
Entropy $\mathcal{S}_T$ ↔ \( \frac{\text{Horizon Area}}{4G_N} \)

Explains results of Bekenstein & Hawking in 1970s.
Ryu & Takayanagi: any subsystem entropy has an associated area!
These subsystem entropies can be defined for any state and quantify entanglement.

Area \( = \frac{S_A}{4G} \)

Area \( = \frac{S_B}{4G} \)

minimal area surface enclosing boundary region corresponding to A
Geometry from Entanglement Structure

\[ |\Psi\rangle \quad \leftrightarrow \quad M |\Psi\rangle \]

subsystem \( A \)

subsystem \( B \)

\[ S_{A_i} \text{ for many } A_i \]

CFT \( \rightarrow \) AdS spacetime decoder

metric \( dS^2(M|\Psi\rangle) \)

* restrictions apply
Constraints on geometries from quantum information theory!

\[ S_A + S_B \geq S_{A \cup B} \]
\[ S_{A \cup B} + S_{B \cup C} \Rightarrow S_B + S_{A \cup B \cup C} \]

constraints on allowed spacetimes
Example:

\[ SS = \frac{1}{\beta} S \langle H \rangle \]

1ST LAW OF THERMODYNAMICS

\[ S \text{Area} \sim \frac{1}{K} S \text{Mass} \]

BLACK HOLE FIRST LAW
\[ \delta S_B = \delta \langle H_B \rangle = \int_B \delta f(x) \langle T_{\mu \nu}(x) \rangle \]

**ENTANGLEMENT FIRST LAW**

Casini, Huerta, Myers

Area of this surface relative to unperurbed spacetime related to asymptotic metric bounded by surface.
\[ \delta S_{B_i} = \delta \langle H_{B_i} \rangle = \int_B \langle T_{00} \rangle \]

**ENTANGLEMENT FIRST LAW**

- **Constraints for all ball-shaped regions:**
  - \(\delta g\) must satisfy Einstein's equations linearized about AdS

**References:**
- Lashkari, McDermott, MVR; Faulkner, Guica, Hartman, Myers, MVR
- Swingle, MVR
Beyond 1st order perturbations?

Need to think carefully about which states we are talking about.

1st order: $S$ in terms of $\langle T_{\mu\nu} \rangle$

$\xrightarrow{\text{local}}$
$\xleftarrow{\text{nonlocal}}$

$\rightarrow$ bulk metric from boundary behavior.

Is there a class of states where $S$ is determined by local data $\langle T_{\mu\nu} \rangle, \langle O_\omega \rangle$?
Euclidean Path Integral States (local sources)

e.g. Skenderis, van Rees; Botta-Cantcheff, Martinec, Silva; Marolf, Porribar, Rabidean, MUR

\[
\langle \phi(x) | \Psi_\lambda \rangle = \int [d\phi(x,\tau)] e^{-S_{Eucl} - \int \lambda(x,\tau) \Theta_a(x,\tau)} \uparrow \text{source} \quad \text{(turn off for } \tau \to 0) \quad \text{local operator}
\]

\[
= \text{Local quantum circuit (non-unitary)}
\]

\[
1\vert \Psi \rangle
\]

\[
\tau
\]

\[
\vdots
\]

\[
1\vert \Psi_0 \rangle \quad \text{start with reference state}
\]

\[
e^{-\epsilon H - \int dx \lambda(x) O_a}
\]
Calculate $S_{\text{Ball}}$ for $|\Psi\rangle_\lambda$ to order $\lambda^2$ in general CFT

* Answer can be reproduced by a geometry
  (using RT if $a^* = c$ otherwise 1-parameter generalization)

* This geometry satisfies Einstein's equations
  to second order about AdS
  (or one-parameter generalization in $a^* \neq c$ case)

Faulkner, Haebl, Hijano
Parrikar, Rabideau, MVR
Can we learn new things about gravity?
Positive energy

\[ \langle \text{M} \rangle \geq \langle \text{vac} | H | \text{vac} \rangle \]

\[ \Delta \langle H \rangle \geq 0 \]

Can define an energy for asymptotically AdS spacetimes and this is always greater than for pure AdS.

**Positive Energy Theorem**
New positive energy theorems:

\[ \Delta \langle H_0 \rangle - \Delta S_B \geq 0 \]

**Positivity of Relative Entropy**

\[ \text{Tr}(\rho \log \rho) - \text{Tr}(\rho_0 \log \rho_0) \geq 0 \]

Can define a gravitational energy for subsystems bounded by extremal surfaces & this energy is positive

**Subsystem Positive Energy “Theorem”**

Lashkari, Lin, Ooguri, Stoica, NYU
More results:

- More general positive energy results from positivity/monotonicity of relative entropy for regions bounded by light cones
  Neuenfeld, Saraswat, MVR
  using Casini, Testa, Torroba

- Deriving gravitational constraints non-perturbatively
  Lewkowykz & Parrikar
  - Use entanglement 1st Law about general states
  - Derive eqn. for evolution of modular Hamiltonian under change in state/region.
Current/future directions I: more general states & quantum effects

Upcoming publication (w. Haehl, Minta, Pollock, Speranza):
- Control structure of bulk entanglement via non-local multi-trace sources

\[ \mathcal{W}(\phi(x)) = \int [d\phi(x,r)] e^{-S_{\text{Euc}} - \int \lambda_\alpha (x,r) \Theta_{\alpha}(r)} - \int \lambda_{\alpha \beta} \Theta_{\alpha}(r) \Theta_{\beta}(r) - \cdots \]

- Related to \[ e^{C_\alpha a_\alpha^\dagger + C_{\alpha \beta} a_\alpha^\dagger a_\beta^\dagger + \cdots} |0\rangle \]

description of bulk state

Can show: \( \Theta(N^2) \) entanglement is exactly the same as for another state with an effective single-trace source.

\[ \lambda_{\text{eff}}(x) = \lambda_\alpha (x) + 2 \int \lambda_{\alpha \beta} (x) \langle \Theta(x) \rangle \lambda_{\text{eff}} + \cdots \]

* Classical results for single-trace states extend to general states above.*
Entanglement at order $N^0$: bulk quantum effects

Does $S_A(1\Xi\{\lambda_i\})$ match with

$$\frac{1}{4G}\langle\text{Area}(\tilde{A})\rangle + S_{\Sigma}^{\text{bulk}}$$

for some $|\psi\rangle_{\text{bulk}}$ on AdS?

Are there novel gravitational effects for matter with exotic entanglement structure?
More questions

- Understand gravitational interpretation of more constraints: e.g.

- Monotonicity of relative entropy
- Strong subadditivity

\[ \lambda \frac{1}{2} \left( \rho_{A_1} \| \rho_{A_1}^{\text{max}} \right) \geq 0 \]

- Extend constraints to include bulk quantum effects

- Extend to non-conformal theories?  
  \( \text{c.f. Faulkner} \)
Is there an entanglement entropy-geometry connection without spatial subsystems?

e.g.: BFSS model (susy matrix QM) in t'Hooft limit?

$P_T$ \[\leftarrow\] extremal surface $\Sigma$

\[\sim 10D \text{ IIA black hole}\]

e.g.: What is \[\frac{d}{dT}(\text{area}(\Sigma))\] in the matrix model?

Entropy of (approximate) subalgebra

w. Anous, Karczmarek, Mintun, Way
To what extent does entanglement entropy probe behind the horizon of black hole microstates?

\[ |\Psi\rangle = N e^{-\alpha H} |1B\rangle \]

Gives:

- Kourkoulou, Maldacena
- Almheiri
- Cooper, Rozali, Swingle
- MVR, Waddell, Wakeham.
Can we reverse engineer quantum systems with entanglement structure matching geometrical features of more general spacetimes?  

see Dong, Silverstein, Tomoba - deSitter?  
tensor network approaches: e.g.

Long term goal: microscopic description of cosmology