Aspects of SYK

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Main research area:
Quantum Gravity

More specifically:
Quantum aspects of black holes
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Quantum Gravity

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Offsprings:  
Quantum thermalization  
ETH  
Random matrices and random systems  
Classical and quantum chaos  
Entanglement and information theory
The problem of finding quantum gravity corrections to black hole dynamics is inextricably mixed with that of finding finite size corrections to thermal behavior

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Even if we know the microscopic theory, this makes it difficult to understand:

- 1/S corrections themselves
- Long time scales
- Long but not so long time scales
- Entanglement and complexity evolution
- Microscopic dynamics
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Vijay Balasubramanian, Jose Barbón, de Boer, Patrick Hayden, Juan Maldacena, Joseph Polchinski, John Preskill, Joan Simon, Leonard Susskind
Sadchev-Ye-Kitaev model:

N Fermions with random k-body couplings

\[ H = \sum_{j < \ldots < l < m < \ldots < n} J_{j \ldots lm \ldots n} c_j^\dagger \cdots c_l^\dagger c_m \cdots c_n \]

\( J_{j \ldots lm \ldots n} \rightarrow \text{Random numbers} \)

(1) Several authors “Embedded random ensembles”
(2) Kitaev: holographic dual, maximal chaos (Maldacena, Shenker, Stanford)
(3) Several authors using Majorana fermions formulation of the model

(2)-Black holes as random particles: entanglement dynamics in infinite range and matrix models. JHEP 1608 (2016)

(3)-Another to appear soon

Correlation between the complexity of the model, parametrized by $k$ (of the $k$-body interaction) and the squared deviations on observables in eigenstates. The complexity of SYK seem to be in the “errors”.

It seems that there is more in quantum thermalization than the originally proposed ETH (Deutsch,Srednicki), which states that deviations are just exponentially suppressed in the entropy.
(2) - Black holes as random particles: entanglement dynamics in infinite range and matrix models. JHEP 1608 (2016)

Quantum quenches in SYK.

Extract the implications of large-N factorization into entanglement dynamics.

Entanglement evolution is extensive at all times, not only at stationarity. Quantitative match to black hole entropy evolution (work with Aron Jansen).

Applications to random tensor networks (HaPPY)? SYK eigenvectors of any k are enough to ensure RT entanglement. We do not need full random vectors.
How microscopic can we go?

Understand implications of unitarity at all times.
\[ |k\rangle \]

\[ k = 1, \ldots, \binom{N}{m} \]
$|k\rangle$

$k = 1, \ldots, \binom{N}{m}$

$p_k(t)$
Diffusion process of exponential complexity...
Effective rate equation provided by detailed balance
Such probability distribution is just the reduced density matrix of the “diagonal operator algebra”

\[
\mathcal{O} = \sum_k \mathcal{O}_k |k\rangle \langle k| \quad \langle \mathcal{O} \rangle = \sum_k \mathcal{O}_k p_k
\]

\[
\rho_D = \sum_k p_k |k\rangle \langle k| \quad S_D = - \sum_k p_k \log p_k
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Which includes all n-point correlation functions of products of number operators...

\[
\mathcal{N}_i \mathcal{N}_j \cdots \mathcal{N}_l
\]
Where is the hope for success?

(1) Maximum decoherence for SYK models. Diagonal Markovian evolution.
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(1) Maximum decoherence for SYK models. Diagonal Markovian evolution.

(2) Each is state is like an ”unstable particle”, decaying to a continuum in the thermodynamic limit.

(3) We use a (not unnoticed) but unexploited symmetry: Permutation or rellabeling symmetry.
Results:

Show that the problem is not exponentially hard but only polynomially hard. Not dependent on the assumption.

The rate equation is solvable. Results for $1/S$ corrections and a new series of “long but not so long” time scales.
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Thanks for listening