LATE TIME BEHAVIOR OF TWO POINT FUNCTIONS IN THE D1D5 CFT

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Late time behavior of two point functions in BH background

- Model: spectral form factor (phase differences are important)
  \[ F_\beta(t) = \sum_{m,n} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t}, \]

- In SYK: early time behavior is governed by GR, late time by RMT [Cotler+many authors, ’16]

- Can we check the validity of the late time prediction in a top-down model?

A zeroth order step: D1D5 system at orbifold point
Time average vs ensemble average in RMT

\[ F_\beta(t) = \sum_{m,n=1}^{2^{10}} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t}, \]

\[ \Delta t = 10 \]
Time average vs ensemble average in RMT

\[ F_\beta(t) = \sum_{m,n=1}^{2^{10}} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t}, \]

\[ F_{\beta=1} \]

\[ \Delta t = 60 \]
Time average vs ensemble average in RMT

\[ F_\beta(t) = \sum_{m,n=1}^{2^{10}} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t}, \]

\[ \Delta t = 110 \]
Time average vs ensemble average in RMT

\[
F_\beta(t) = \sum_{m,n=1}^{2^{10}} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t},
\]

\[
\Delta t = 160
\]
Time average vs ensemble average in RMT

Progressive average:

\[ \Delta t = at \]

- No a priori knowledge of time scale is required
- Even spacing on log-log plots...
Time average vs ensemble average in RMT

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Time average vs ensemble average in RMT

Progressive average:
\[ \Delta t = at \]

Dependence on \( a \)?
Two charge black holes in D1D5

D1D5 system: \( CFT_N = \frac{(T^4)^N}{S_N} \), \( N = N_1N_5 \)

- IR description of 5d black holes
- Dual to IIB on \( AdS_3 \times S^3 \times T^4 \)

Ramond ground states

\[
|N_{n\mu}, N'_{n\mu}\rangle = \prod_{n,\mu} (\sigma_n^\mu)^{N_{n\mu}} (\tau_n^\mu)^{N'_{n\mu}} |0\rangle
\]

\[
\sum_{n,\mu} n(N_{n\mu} + N'_{n\mu}) = N, \quad N_{n\mu} = 0, 1, 2, \cdots, \quad N'_{n\mu} = 0, 1
\]

- Parametrically large, but not classically visible degeneracy \( S \sim \sqrt{N} \)
- Typical state from grand canonical distribution \( e^{-\eta N} \)

\[
N_{n\mu} = \frac{1}{e^{\eta n} - 1}, \quad N'_{n\mu} = \frac{1}{e^{\eta n} + 1}, \quad N_n = \sum_{\mu} (N_{n\mu} + N'_{n\mu}) = \frac{8}{\sinh \eta n}
\]

\[
N = \sum_n nN_n \approx \frac{2\pi^2}{\eta^2}
\]
Two point function in D1D5

Two point function [Balasubramanian,...’05]

\[ \mathcal{O} = \frac{1}{\sqrt{N}} \sum_{a=1}^{N} \mathcal{O}_a, \quad h_a = \bar{h}_a = 1. \]

\[ G(w, \bar{w}) = \langle N_{n\mu}, N'_{n\mu} | \mathcal{O}^\dagger \mathcal{O} | N_{n\mu}, N'_{n\mu} \rangle \]

\[ = \frac{1}{N} \sum_{n=1}^{N} nN_n \sum_{k=0}^{n-1} \frac{1}{\left[ 2n \sin \left( \frac{w-2\pi k}{2n} \right) \right]^2 \left[ 2n \sin \left( \frac{\bar{w}-2\pi k}{2n} \right) \right]^2}. \]

Remove light crossing singularities: divide by vacuum 2pt function

\[ \hat{G}(w, \bar{w}) = \frac{1}{N} \sum_{n=1}^{N} nN_n \sum_{k=0}^{n-1} \left( \frac{4 \sin \frac{w}{2} \sin \frac{\bar{w}}{2}}{2n \sin \left( \frac{w-2\pi k}{2n} \right) 2n \sin \left( \frac{\bar{w}-2\pi k}{2n} \right)} \right)^2. \]
Two point function in D1D5
Two point function in D1D5
Two point function in D1D5

\eta = 0.05 + 0.025j, \quad j = 0, \ldots, 10
Two point function in D1D5

Analytic understanding for the late ramp:

\[ R(t) = \frac{1}{N} \sum_{n=1}^{t/\gamma} \frac{8}{\sinh(\eta n)} + \frac{1}{N} \sum_{n=1}^{t/(2\gamma)} \frac{1}{2} \frac{8}{\sinh(2\eta n)} \]

\[ \approx \frac{5\sqrt{2}}{\pi \sqrt{N}} \log \left[ \frac{\sqrt{N}}{\sqrt{2\pi \delta}} \tanh \left( \frac{t\pi}{\sqrt{2N\gamma}} \right) \right] + \frac{8\eta}{\sqrt{2N\pi}} \log 2 \]
Two point function in D1D5

Scales involved naturally differ from RMT

\[ t_p \sim \sqrt{N} \]

\[ H_{\text{plateau}} \sim \frac{\log \sqrt{N}}{\sqrt{N}} \]

Ramp is not linear but still seems to be parametrically long (persist for \( N \to \infty \))

Intuition from progressive time average:

- \( t_d \): inverse of the largest level spacing
- \( t_p \): inverse of the smallest level spacing

At finite coupling we then anticipate

- \( t_d \): no change
- \( t_p \): grows to \( e^S \)
Questions?