Nonlinear Gravity From Entanglement

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Simons It From Qubit Collaboration Annual Meeting

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Setup: states $|\Psi(\lambda)\rangle$ of CFT $|\Psi(0)\rangle = |\text{vac}\rangle$

Assume: entanglement entropy $S_B$ for ball-shaped regions computed from geometries $M(\lambda)$ via the H.R.T. formula

$S_B = \frac{\text{Area}(B)}{4G_N} + ...$
$\lambda = 0 \quad |\psi\rangle = |\text{vac}\rangle$

For any CFT:

$$M = \text{AdS}_{d+1}$$

corresponds entanglement entropy for ball.

* holographic CFT: HRT in AdS will also give right answer for other regions.*
\[ \mathcal{O} (a) : \]

Have \[ SS_B = S \left< -\log \rho_B \right> \]

CFT: \[ \rho_B = e^{-H_{\xi \mathcal{B}}} \]

\[ SS_B = \int_B f(x) \cdot S \left< T_{\mu \nu}(x) \right> \equiv SE_{\xi \mathcal{B}} \]

Entanglement First Law

\[ \text{first law of thermodynamics} \]

Blanco, Casini,Hung, Myers

density matrix for ball-shaped region in vacuum state.

energy density

first law of thermodynamics
$O(\lambda)$:

$SS_B = \int_B f(x) \langle T_{00}(x) \rangle$

Entanglement First Law

$\frac{1}{z^d-2} S g_{1}^{1/2} \mu(x, z=0) \propto \langle T_{\mu\nu}(x) \rangle_{\text{CFT}}$

Holographic Dictionary for stress tensor

$S g_{1}$ satisfies Einstein's Equations, linearized about AdS. w. Lashkari McDermott
Use magic Hollands/Wald identity = Stokes Thm.

\[ \frac{d}{d\lambda} (\text{Area}_{\bar{B}} - E_{\xi_{\bar{B}}}) = \int_{\Sigma} \omega (g, \frac{d}{d\lambda} g, \xi g) \]

\[ + \int_{\Sigma} \text{Einstein} (g, \frac{d}{d\lambda} g) \]

\[ \downarrow \mathcal{O}(\lambda) \]

\[ \frac{d}{d\lambda} (S_{\bar{B}} - E_B) = \int \xi^a E_{(i)}^{ab} \xi^b \]

\[ \text{vanishes by first law+HRT} \]

\[ \text{Einstein tensor} \]

\[ \xi_{\bar{B}}: \text{boundary conformal Killing vector} \]

\[ \xi: \text{bulk vector vanishing at} \ \bar{B} \]

w. Faulkner, Guica, Hartman, Myers
Can we say anything at the non-linear level?
Have inequalities: for any \( |\Psi\rangle \)

\[
\Delta S_B \leq \Delta \int_B f(x) \langle T_{oo}(x) \rangle
\]

CFT Interpretation: RELATIVE ENTROPY is positive

\[ S(\rho_\Sigma || \rho_B) \geq 0 \]

Gravity Interpretation: \( H_\Sigma \geq 0 \)

*Positive energy theorem for subsystems*

w. Lashkari, Lin, Ooguri, Stoica
Can we actually get nonlinear Einstein's equations?

Idea:

First order E.E. from

\[ S^\lambda_B = \int f(x) \langle T_{\lambda\nu} \rangle \]

+ H.R.T. + Wald/Hollands

Maybe nonlinear corrections to E.E. come from corrections to this entanglement formula.
What do these corrections look like?

Start by working backwards:

Assuming Einstein's eqns:

\[ \langle O_\alpha \rangle_{\text{CFT}} \rightarrow \text{asymptotic bulk fields} \]

\[ T_{\mu \nu}, \theta_0, S, \text{etc...} \]

bulk fields in interior

\[ \downarrow \text{perturbative E.E.} \]

area of extremal surface

\[ \downarrow \text{Wald technology} \]

entanglement entropy

\[ \downarrow \text{H.R.T.} \]

\[ S = S_0 + \lambda \int_B f(x) \langle T_{00} \rangle + \lambda^2 \int \int K_{\alpha}(x, y) \langle O_\alpha(x) \times O_\alpha(y) \rangle + ... \]

W. Beach, Lee, Rabideau
Now try to reverse steps.

Can we derive this:

$$S = S_0 + \lambda \int_{\mathcal{B}} f(x) \langle T_{00} \rangle + \lambda^2 \int \int K_\alpha (x, y) \langle \Theta_\alpha (x) \Theta_\alpha (y) \rangle + \ldots$$

from CFT?

1st order: UNIVERSAL

2nd order: likely need to make assumptions about STATE and THEORY

Einstein's eqns relevant to "holographic states" in holographic theories
Choice of state:

\[ \langle \phi_0 | \text{vac} \rangle = \int [d\phi] \ e^{-S_{\text{Eucl}}} \phi(0) = \phi_0 \]

\[ \langle \phi_0 | \Sigma_\lambda \rangle = \int [d\phi] e^{-S_{\text{Eucl}} - \int_{t < 0} \lambda_\alpha(x)(2t)\Theta_\alpha(x)} \]

\( \lambda_\alpha \rightarrow \) Euclidean bulk solution

Initial data for Lorentzian solution

Lorentzian solution to one-pt fns.

Gives coherent state of bulk fields.

w. Marolf, Parrikhari Rabiden
Starting from

\[ |\Phi_\lambda\rangle = \int [d\phi] e^{-S_{\text{Eff}} - \int \lambda_\alpha \phi_\alpha} \]

Compute \( S_B \) to 2nd order in \( \lambda_\alpha \)

c.f. Faulkner, Sperenza, Parrikar, Leigh, Wang

**Result depends on:**

\[ \langle \phi_\alpha \phi_\beta \rangle, \langle \phi_\alpha \phi_\beta T_{\mu\nu} \rangle, \langle T_{\mu\nu} T_{\alpha\beta} \rangle \rightarrow \text{universal (up to normalization)} \]

\[ \langle T_{\mu\nu} T_{\alpha\beta} T_{\rho\sigma} \rangle \rightarrow 3 \text{ different tensor structures} \]
Preliminary results:

Hollands Wald:

\[ \frac{d^2}{d\lambda^2} (\text{Area}_B - E_{\xi_B}) = \int \omega (g, \delta g, \xi, \delta g^{(\ast)}) \]
\[ - \frac{1}{8\pi} \int \xi^a E_a^{(2)} \xi^b \]

CFT calculations:

\[ \frac{d}{d\lambda^2} (S_B - \langle H_{\xi_B} \rangle) = \int \omega (g, \delta g, \langle \xi \rangle, \delta g^{(\ast)}) \]
\[ - \int \xi^a \xi^b T^{(2)}_{ab} (\langle \Theta \rangle_{\text{matter}}, \langle \Theta \rangle_{\text{matter}}) \]

Combine:

\[ E^{(2)}_{ab} = 8\pi \cdot T^{(2)}_{ab} \]

For correct \( \langle TTTT \rangle \) and "holographic" states
**Summary**

- CFT with appropriate $\langle T T T \rangle$
- $|\Psi(\lambda)\rangle_{\text{CFT}}$ from light operator sources in Euclidean P.I.
- Assume $S_{\text{Ball}}(|\Psi(\lambda)\rangle)$ computed from $M_\lambda$ via HRT

\[ M(\lambda) \text{ must satisfy Einstein's eqns to 2nd order in } \lambda \text{ w. boundary conditions given by CFT one point fps } \langle \Theta \rangle, \langle T \rangle \]