Spontaneously Ordered Electronic States in Graphene

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Simons Symposium: Quantum Physics Beyond Simple Systems
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New ordered states in SLG and BLG

- Weak interactions in undoped SLG (low DOS) both a blessing and a curse: robustness vs. functionality
- Strengthen the effects of interaction: use weakly dispersing states, $E_{\text{kinetic}} < E_{\text{potential}}$

(i) SLG doped to saddle point: chiral d-wave superconductivity (broken time-reversal symmetry)

(ii) BLG at charge neutrality: excitonic insulator, spontaneous Hall effect at $B=0$ [charge QHE, spin QHE or valley QHE], nematic order

(iii) alter electronic states using external fields (QHE, FQHE)

- Ways to experimentally distinguish different ordered states in BLG
Electronic states in strongly doped graphene

- Quadratic dispersion near saddle points at $E=+t_0,-t_0$
- Logarithmic Van Hove singularity
- Hexagonal FS @ $n=3/8,5/8$
- Similar to square lattice @ $n=1/2$
- Various competing orders: CDW, SDW, superconductivity, nematic order (Pomeranchuk instability)

High doping required ($\delta n=1/8$)
Electrostatic gating challenging
Can be achieved chemically (Berkeley) or with liquid dielectric gating (Columbia, Geneva)
Different scenarios

- Nesting and vH singularity enhance interaction effects
- d-wave pairing, Kohn-Luttinger framework (Gonzalez 2008)
- Pomeranchuk (nematic) order, mean field (Valenzuelo, Vozmediano 2008)
- SDW order, mean field (Li arxiv:1103.2420, Makogon et al arxiv:1104.5334)
- Legitimate mean-field states: superconductor, metal, insulator
- Need renormalization group (RG) to compare these orders on equal footing
Attraction from repulsion

* Approach developed for square lattice


* RG treats all potential instabilities on equal footing

* Progressively integrate out high energy states, examine flow of couplings

* Marginal with log corrections

* Three sources of log divergences: DOS, BCS, nesting

\[ L = \sum_{\alpha=1}^{3} \psi_\alpha^+ \left( \partial_t - \epsilon_k + \mu \right) \psi_\alpha - H_{\text{two-particle}} \]

* Pairing interaction induced by spin fluctuations

* New scenario for the competition of SDW and SC
Low energy description: three inequivalent patches

\[ L = \sum_{\alpha=1}^{3} \psi_\alpha^+ (\partial_t - \epsilon_k + \mu) \psi_\alpha - H_{\text{two\--particle}} \]

four interactions
(i) marginal at tree level
(ii) log's

Nandkishore, Chubukov & LL Nat Phys (22 January 2012)
Chiral superconductivity from repulsive interaction

- Pairing gap winds around the Fermi surface
- Induced by (weak) repulsive interactions
- $d$-wave pairing wins over $s$-wave pairing
- $d+id$ state: \textit{time reversal symmetry broken}
- Once a candidate for high $T_c$, long abandoned
  - (i) nonzero Chern class ("charge QHE" at $B=0$);
  - (ii) spin and thermal QHE; edge charge current in $B$ field
  - (iv) Majorana states @ vortices and boundaries
  - (v) Kerr effect, interesting Andreev states, etc
Two-particle inter- and intra-patch scattering processes

\[ H_{\text{two-particle}} = \sum_{\alpha, \beta=1}^{3} \left( \frac{g_1}{2} \psi_{\alpha}^+ \psi_{\beta}^+ \psi_{\alpha} \psi_{\beta} + \frac{g_2}{2} \psi_{\alpha}^+ \psi_{\beta}^+ \psi_{\beta} \psi_{\alpha} + \frac{g_3}{2} \psi_{\alpha}^+ \psi_{\alpha} \psi_{\beta} \psi_{\beta} \right) + \sum_{\alpha=1}^{3} \frac{g_4}{2} \psi_{\alpha}^+ \psi_{\alpha}^+ \psi_{\alpha} \psi_{\alpha} \]
Diverging susceptibilities

SC pairing (spin-up, spin-down)

\[ \Pi_{pp}(0) = \frac{\nu_0}{4} \ln \frac{\Lambda}{\max(\mu, T)} \ln \frac{\Lambda}{T} \]

SDW susceptibility

\[ \Pi_{ph}(Q_i) = \frac{\nu_0}{4} \ln \frac{\Lambda}{\max(\mu, T)} \ln \frac{\Lambda}{\max(\mu, T, t_3)} \]

Lesser susceptibilities:

\[ \Pi_{pp}(Q_i), \Pi_{ph}(0) = \frac{\nu_0}{4} \ln \frac{\Lambda}{\max(\mu, T)} \]

Imperfect nesting
RG flow of couplings for n patches

\[
\frac{dg_1}{dy} = 2d_1 g_1 (g_2 - g_1) \quad \frac{dg_2}{dy} = 2d_1 (g_2^2 + g_3^2) \quad \frac{dg_4}{dy} = -(n-1) g_3^2 - g_4^2
\]

\[
\frac{dg_3}{dy} = -(n-2) g_3^2 - 2g_3 g_4 + 2d_1 g_3 (2g_2 - g_1)
\]

RG time \( y = \ln^2 \xi = \Pi_{pp} \)

Nesting parameter \( d_1 = \frac{d \Pi_{pp} (Q)}{d \Pi_{pp} (0)} < 1 \)

Critical couplings \( g_i (y) \approx \frac{G_i}{y_c - y} \)

Initial values \( g_i (y=0) \approx 0.1 \)
**RG flow features**

- Agrees with the square lattice (n=2)
- Unique fixed trajectory (“stable fixed point”) for repulsive bare couplings
- $g_1$, $g_3$, $g_2$ cannot change sign, stay positive
- $g_4$ decreases & reverses sign
- $g_3$-$g_4$ large & positive, drives SC instability
- Positive $g_3$ penalizes s-wave, favors d-wave SC
- Susceptibility $\chi_{sc}$ diverges faster than $\chi_{sdw}$
- **SC a clear winner** (cf. square lattice)
- High $T_c$ from weak coupling physics

$$T_c \approx \Lambda e^{-\frac{A}{\sqrt{g_0 \nu_0}}}$$
Competition of d-wave orders below $T_c$

- By symmetry, two degenerate d-wave states
- Ginzburg-Landau analysis of competition

$$\Delta = \Delta_a (x^2 - y^2) + \Delta_b 2xy$$

$$F(\Delta_a, \Delta_b) = \alpha (T - T_c)(|\Delta_a|^2 + |\Delta_b|^2) + K_1 (|\Delta_a|^2 + |\Delta_b|^2)^2$$

$$+ K_2 |\Delta_a^2 + \Delta_b^2|^2$$

- Calculation of GL functional yields $K_2 > 0$
- d+id and d-id ground states $\Delta_a = \pm \Delta_b$
- Superconductivity with TRS breaking
Summary: chiral SC in doped graphene

- Interaction driven instability in graphene doped at saddle points
- Weak repulsive interaction stabilizes chiral superconducting state $d + id$ or $d-id$
- Enhanced $T_c$
- Topological superconductor with broken TRS
Outlook:

- Topological superconductor with broken TRS
- Zoo of interesting phenomena
- Higher-genus fullerens
- Graphene easily combined with other materials into hybrid structures and heterostructures: pathway to applications of chiral superconductivity

[Diagram of higher-genus fullerens and point junctions]
Spontaneously ordered states in bilayer graphene

\[ \hat{H} = -\frac{\hbar^2}{2m} \begin{pmatrix} 0 & (k_x - ik_y)^2 \\ (k_x + ik_y)^2 & 0 \end{pmatrix} \]

F Wang (LBL)

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Bilayer at charge neutrality (no disorder, no trigonal warping)

- Finite DOS at $\varepsilon=0$ (quadratic dispersion)
- Fermi surface reduced to a point
- Fermi liquid unstable due to interband transitions
- Log-divergent 2-particle interaction vertices, self-energy, effective mass, etc
- RG similar to g-ology in $D=1$
Non-Fermi liquid even at weak interaction: Greens function log^2 renormalization

\[ G(\omega, k) = \frac{Z(\xi)}{i\omega - H_0(k)}, \quad \xi = \ln \frac{\Lambda_0}{\sqrt{\omega^2 + (k^2/2m)^2}} \]

\[ V_{RPA}(\omega, k) = \frac{2\pi e^2}{\kappa k - 2\pi e^2 N \Pi(\omega, k)} \]

RG flow at log^2 order (Nandkishore & LL 2010)

\[ \frac{\partial Z}{\partial \xi} = -\xi \frac{2Z(\xi)}{N \pi^2} \quad N = 4 \]

\[ G(\xi) = A G_0(\omega, k) \exp \left( -\xi^2 / N \pi^2 \right) \]

\[ \Sigma \sim \xi^2 \left( i\omega - H_0(k) \right) \]

Compare with the diffusive Coulomb Anomaly (Altshuler, Aronov, Lee 1980)

\[ \frac{\partial Z}{\partial \xi} = -\frac{\xi}{4\pi^2 g} Z(\xi), \quad \omega \tau \ll 1 \]

\[ \delta m = \frac{0.56 \xi}{2N \pi \ln 4} m_0 \approx 0.016 \xi m_0 \quad \text{RG for interaction, see Falko's talk} \]

2D conductance

\[ \kappa = 2.5 \]

\[ \kappa = 1 \]

\[ E > 0 \]

\[ E = 0 \]
Theory:

Experiment:


Spontaneous ordering in BLG at DP

- Particle-hole pairing instability
- BCS-like exciton condensate, no superfluifity, phase locking
- Gapped spectrum $\Delta = \pm \Delta_0$
- Another candidate state: “nematic” order, gapless spectrum, broken rotational symmetry

Vafek, Yang 2010; Lemonik et al 2010

Min et al 2008; Nandkishore, LL 2010
Zhang et al 2010

Simons symposium: QP beyond simple systems
Spontaneous gap opening in BLG


- 'Which-layer' symmetry breaking
- Domains of + and – polarization
- Charge, valley or spin polarized current along domain boundaries, QHE, VQHE, SQHE, etc
- SU(4) symmetry and the variety of possible states
- Time reversal symmetry breaking at B=E=0: Anomalous Quantum Hall state, quantized $\sigma_{xy}$
- Experiment (Yacoby, Lau and Geim groups)

Velasco et al arXiv:1108.1609
Large variety of possible states

$$H_K = \begin{pmatrix} \Delta_K & p_-^2/2m \\ p_+^2/2m & -\Delta_K \end{pmatrix} \quad H_{K'} = \begin{pmatrix} \Delta_{K'} & p_+^2/2m \\ p_-^2/2m & -\Delta_{K'} \end{pmatrix}$$

$$\Delta_K, \sigma = \pm \Delta_{K'}, \sigma = \pm \Delta_K, -\sigma = \pm \Delta_{K'}, -\sigma \quad p_\pm = p_1 \pm ip_2$$

- Four-fold spin/valley degeneracy
- Degeneracy on a mean field level: instability threshold **the same for all states**: short-range interaction, screened long-range interaction models
- SU(4) symmetry?
Opposite chirality of two valleys conceals SU(4) symmetry, made manifest by performing unitary transformation

\[ H_0 = \frac{p_+^2}{2m} \tilde{\tau}_- + \frac{p_-^2}{2m} \tilde{\tau}_+, \]

Approximate SU(4) symmetry (weakly broken by trigonal warping and capacitor energy)

\[ H = \sum_p \psi_p^\dagger H_0 \psi_p + \frac{1}{2} \sum_q V_+(q) \rho_q \rho_{-q} + V_- \lambda_q \lambda_{-q}, \]

Strategy: Diagonalize SU(4) invariant Hamiltonian and incorporate anisotropies perturbatively
Mean field description of gapped states

\[ H = \frac{p_+^2 \tau_- + p_-^2 \tau_+}{2m} + \Delta \tau_3 Q. \]

Classification into manifolds (4,0), (3,1), (2,2) and distinction between symmetry protected and accidental degeneracies

\[ \sigma_{xy} = (M_\uparrow - M_\downarrow) \frac{e^2}{h}, \]

(4,0) and (3,1) states feature QHE, "anomalous QHE", B=0

Nandkishore, LL 2010   Vafek, Yang 2010

**Near-degeneracy and selection:**
Quantum fluctuations favor (4,0) state;
thermal fluctuations favor (2,2) state
The QAH state stabilized by a B field

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<th>Energy</th>
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(a) QAH state $B \Delta < 0$ $\nu = 4$
(b) QAH state $B \Delta > 0$
(c) QSH state

$n = \pm 4 \times eB/h$

Theory: the QAH state favored at small nonzero B and $\nu = +4, -4$

Experiment: Incompressible $\nu = +4, -4$ states observed at very low B

Consistent with the QAH state

Martin et al (2010)
Mayorov et al (2012)
Transport experiments compatible with the QAH state (but indecisive)

- Incompressible regions at low $B$, $\nu=4$ (if field induced), $\nu=+4$ and $\nu=-4$ (if intrinsic); no such feature at higher filling factor (unlike nematic or other states)

- Incompressible (bulk gap)+finite two-probe conductivity; distinguishes QAH state from (2,2) state but not from nematic state or trigonal warping

- Phase transition at zero $\nu$, finite $B$ to (2,2) QHFM state (likewise)

- Phase transition at finite $E$ to trivial insulator (Ising universality class)

The QAH state not yet observed
1) Direct test: measurement of QHE at B=0; requires four-probe measurement on suspended BLG at low T

2) TRS breaking via violation of Onsager symmetry $B,-B$ in a four-probe measurement

3) Optically detect TRS breaking: contactless measurement of $\sigma_{xy}$ by polar Kerr effect (not Faraday effect)

Nandkishore & LL, PRL 107, 097402 (2011)

4) Scanning photocurrent imaging: domains with different chirality, p-n droplets, edge states


5) Tunneling probes and local capacitance probes: local gap, filling factor, compressibility


Kerr effect: optical detection of TRS breaking, contactless measurement of $\sigma_{xy}$

Large polar Kerr effect in TRS-broken states: interband transitions sensitive to low-energy physics at Dirac point

Nandkishore & LL
PRL 107, 097402 (2011)

Scanning photocurrent (PC) imaging


\[ j_{local} = (a \nabla n + b \hat{z} \times \nabla n) J_{laser} \]

- Unpolarized light generates PC at interfaces, inhomogeneities, edges
- PC can image domains of opposite chirality, p-n boundaries, etc
- How are local properties manifested in system-wide PC?
System-wide (global) PC in gapless materials: imaging local properties

Essential nonlocality and directional effect

Electrostatic analogy
$h$ dependence?
System-wide (global) PC in gapless materials: imaging local properties

Essential nonlocality and directional effect

Electrostatic analogy

\[ q' = -\frac{h}{d}q \]
\[ q'' = -\frac{d-h}{d}q \]
\[ q'_{\text{dipole}} = -\frac{p_z}{d} \]
\[ q''_{\text{dipole}} = \frac{p_z}{d} \]

h-independent!
Nonlocality and directional effect
“Shockley-Ramo theory”


Angle-dependent global response, no position dependence
Tunneling heterojunctions in BLG: domains with different stacking order

Features:

• Tunneling transport (depends on orientation)
• New tunneling probe of ordered states
• Energy-dependent conductance, suppressed near DP (can mimic/obscure gapped state)
BLG summary

- Rich pattern of phases, SU(4) classification
- Possibility of realizing QAH state at low T
- Inducing QAH state with B field
- **Experimental verdict**: QAH order plausible, but more work needed
- Additional experimental probes: optical Kerr effect, photocurrent imaging, tunneling
Collaboration
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