This symposium focused on the current state of the art and most promising directions for future development in areas related to tropical and nonarchimedean analytic geometry, with a special emphasis on relations to birational geometry and the minimal model program (MMP). Recent results related to interacations between nonarchimedean geometry and MMP that we discussed in detail include:

1. The work of Mustata–Nicaise on connectedness of Kontsevich–Soibelman skeletons for varieties with semi-ample canonical divisors, using a generalization of relative Kawamata-Viehweg vanishing to families over a valuation ring.

2. The results of de Fernex–Kollar–Xu on dual complexes for dlt pairs with applications to contractibility for dual complexes of log terminal singularities and rationally connected degenerations, by explicitly keeping track of dual complexes while running a carefully chosen MMP.

3. The proof by Brown–Foster that analytifications of morphisms with rationally connected fibers are homotopy equivalences.

4. The work of Nicaise–Xu showing that essential skeletons of varieties with semi-ample canonical divisors are skeletons in the stronger sense that they are deformation retracts of the analytic space.

Many intriguing open problems remain, including:

1. The extension of such results to positive and mixed characteristic, and to higher dimensional base rings such as $k[[t_1, t_2]]$, where MMP is not yet established and there are additional difficulties with vanishing theorems.

2. The question of whether essential skeletons of totally degenerate Calabi–Yau varieties are homotopic to spheres (the answer is affirmative up to dimension three, relying on results mentioned above together with Perelman’s proof of the Poincare Conjecture).

3. The existence of a “Morse flow” toward the essential skeleton (which would somehow realize the MMP within the context of non-Archimedean analytic geometry).

Another important theme highlighted during the symposium is the interaction between nonarchimedean analytic and complex analytic methods and results, especially in relation to solutions to partial differential equations and the geometry of currents. Specifically:

1. Gubler presented joint work with K"unnemann extending the work of Chambert-Loir–Ducros on forms and currents on skeletons of Berkovich spaces (which was discussed extensively at the 2013 symposium), to include currents associated to tropical subvarieties. This new theory of Gubler–K"unnemann $\delta$-forms and $\delta$-currents features rich interactions between tropical geometry and Arakelov theory with the potential for applications to diophantine geometry.

2. Boucksom and Jonsson presented their joint work with Favre solving a non-Archimedean analogue of the Monge–Ampere equation, a story which began with the work of Kontsevich–Soibelman on homological mirror symmetry and the subsequent foundational work of Kontsevich–Tschinkel. Rich interactions between Berkovich spaces and birational geometry also emerge in this setting, with multiplier ideals and vanishing theorems playing a key technical role in the proofs.

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(3) Kedlaya spoke on convergence polygons for connections on curves, a topic of intense recent activity which provides a conceptual explanation for and a vast generalization of Dwork’s theory of radii of convergence for solutions to $p$-adic differential equations.

(4) Temkin discussed metrization of pluricanonical forms in the nonarchimedean setting, which gives a new approach to the theory of essential skeletons discussed above, formulated in a purely analytic way. (For the record, Temkin has submitted his groundbreaking manuscript on this topic to the proceedings of these Symposia.)

(5) Huh announced an important breakthrough, joint with Babaee, giving counterexamples to Delauny’s “strongly positive Hodge conjecture” via complex currents associated to tropical varieties and initiating a new strategy for disproving the Hodge conjecture itself.

These are just two of the important themes running through the symposium. Other topics covered include:

(1) The proof by Ducros and Thuillier that skeletons, as introduced by Berkovich and Thuillier, possess a canonical integer-affine structure. This relies on some results in model theory in addition to the concept of “reified valuations” introduced recently by Kedlaya.

(2) Recent advances of Gubler–Rabinoff–Werner extending work on faithful tropicalizations of curves by Baker–Payne–Rabinoff to higher dimensional varieties.

(3) Applications of tropical methods to Brill–Noether theory in ongoing work by Jensen and Payne, including a proof of the Maximal Rank Conjecture for quadrics.

(4) New results related to Igusa integrals, including the work of Chambert-Loir–Tschinkel on volume asymptotics and the recent solution by Nicaise–Xu of Veys’s conjecture on poles of maximal order for Igusa zeta functions.

(5) The existence of integral étale cohomology groups of compact strictly $K$-analytic spaces, constructed by Berkovich via complex analytic vanishing cycles for formal schemes, building on Temkin’s desingularizations of quasi-excellent schemes in characteristic zero, constructions of Kato and Nakayama in log geometry, as well as his own prior work on vanishing cycles for formal schemes.

In addition to the lectures and informal discussion periods, the symposium included two moderated hour-long problem sessions, in which participants presented compelling open problems related to the themes of the symposium. These sessions were carefully documented and notes with statements of the problems and a brief account of the surrounding discussions are available on the symposium website. We hope that these notes will serve as guidance and motivation for young researchers hoping to work in this exciting and rapidly developing field. Nearly all of the speakers have written up lecture notes for dissemination on the symposium website, and many have also submitted expanded versions of their lecture notes as survey articles for the upcoming proceedings volume.

There seemed to be widespread agreement among the participants that the interaction between experts in birational geometry (Abramovich, Mustata, Xu,...), algebraic geometry (de Jong, Vakil,...), non-Archimedean analytic geometry (Berkovich, Temkin, Chambert-Loir, Ducros,...), tropical geometry (Rabinoff, Werner), complex geometry (Boucksom, Huh,...), and Arakelov geometry (Gubler, Künemann,...) was unprecedented and highly productive.