Galaxy Scaling Relations in LCDM

Julio F. Navarro
The Clustering of Dark Matter
The Millennium Simulation Series

Simulations have enabled a full characterization of the (hierarchical) clustering of cold dark matter on large and small scales.

- Dark matter halos are self-similar in structure
- Mass function well constrained and understood

Dark matter halos in LCDM
The self-similar nature of LCDM halos

DM halos: self-similar structures linked by the age of the Universe

$10^{12} \, M_{\odot}$

$10^{15} \, M_{\odot}$

$M/R^3 = \text{constant}$
The self-similar nature of LCDM halos

\[ M_{200}/r_{200}^3 = \text{constant} \]

\[ M_{200} \propto V_{200}^3 \]

acceleration: \[(V_{200}^2/r_{200})/(cH_0) = 10*(V_{200}/c) = 0.01*(V_{200}/300 \text{ km/s})\]
The shape of the mass profiles of dark matter halos is roughly independent of halo mass and cosmological parameters.

Density profiles are “cuspy”

$$\frac{\rho}{\rho_{\text{crit}}} = \frac{\delta_c}{(r/r_s)(1+r/r_s)^2}$$
\[ \frac{\rho}{\rho_{\text{crit}}} = \frac{\delta_c}{[(r/r_s)(1+r/r_s)^2]} \]

- At fixed mass, the only radial scale is given by the scale radius, \( r_s \).
- The \( \rho \propto r^{-1} \) central cusp implies **constant acceleration**: galaxies form in regions where the DM acceleration varies little with radius.

\[ a \propto GM(r)/r^2 \]

\[ a_{\text{max}} = a(0) = \text{const} \times V_{200}^2/r_{200} \]

\[ a_{\text{max}}/(cH_0) = (1/10)(V_{200}/300 \text{ km/s}) \]
CDM predicts a single mass/circular velocity profile for a given velocity scale.

"Rotation curves" would be rising, not flat, without the contribution of baryons.

$V_{\text{max}}$ and $r_{\text{max}}$ are another way of specifying the mass/size of the halo.

Oman+15
Galaxy Scaling Laws in LCDM

- Halo mass-galaxy mass relation
- Galaxy mass-size relation
- Tully-Fisher relation
- Mass discrepancy-acceleration relation
• CDM halo mass function is now well understood in all mass scales relevant to galaxy formation.
CDM halo mass function vs galaxy luminosity function

- CDM halo mass function much steeper than the galaxy luminosity function at the faint end.
- Reconciling the two requires a highly non-linear dependence between galaxy and halo mass.
- At low masses reionization, as well as feedback from evolving stars, are thought to be responsible.
- Most dwarf galaxies live in halos of the same mass. Galaxy formation efficiency should decline in low mass halos.

Benson+2000
Abundance Matching: Galaxy Stellar Mass vs Halo Mass

- Galaxy formation efficiencies are very low, peak at 15% for Milky Way-like galaxies.
- Steep dependence at low halo mass—a fundamental result of galaxy formation models.
- Most dwarfs form in halos of similar mass and hence similar properties—what is the origin of their diversity then?
- Very few luminous galaxies should form in halos with mass below a “threshold” of $10^{10} \, M_{\odot}$.
Large hydrodynamical simulations of cosmologically representative volumes (~100 Mpc box) have recently been completed (see; e.g., results from the Illustrius Project).
Recent simulations have been able to include reionization and feedback effects to reproduce the galaxy stellar mass function down to galaxies of $M_* \sim 10^8$ solar masses in stars.

They also match reasonably well other properties of the observed galaxy population.
Galaxy mass-halo mass scaling is essentially that of abundance matching.
Galaxy mass-size relation parallels the halo mass-size relation.

Navarro+17
The Tully-Fisher relation
A power-law scaling relating a disk galaxy’s luminosity (stellar mass) with its rotation speed

- A powerful secondary distance indicator

Velocity width $\sim 2V_{\text{rot}}$
The Tully-Fisher relation

- Zero-point and slope difficult to reproduce in early cosmological simulations

Navarro & Steinmetz 2000
The Tully-Fisher relation

Galaxies too massive and centrally concentrated to give “flat rotation curves”

Zero-point and slope difficult to reproduce in early cosmological simulations

Stellar mass
The Tully-Fisher relation in LCDM

- TFR does not just reflect the mass-velocity ($M \propto V^3$) scaling of dark matter halos.
- Galaxy masses are not proportional to halo masses. Rotation velocities are a function of the galaxy mass, size, and the dark matter contribution within the galaxy half-mass radius.
- Need galaxies that are important gravitationally, and halos that “contract” as a result of galaxy assembly.

Ferrero+2016
Need galaxies that are important gravitationally (need to have the right size), and halos that “contract” as a result of the galaxy assembly.

At fixed halo mass, galaxies that are more massive than the average rotate faster than the average, and vice versa—the scatter spreads along the Tully-Fisher relation, leading to small dispersion in the relation.

Ferrero+2016
The Tully-Fisher relation in EAGLE

- The resulting relation is in excellent agreement with the observed Tully-Fisher relation, including its redshift evolution

Ferrero+2016
The Mass Discrepancy-Acceleration relation (MDAR)
Disk galaxy rotation velocities may be “predicted” from the distribution of luminous matter

\[ g_{\text{obs}} = \frac{V^2(R)}{R} \]

The mass discrepancy-acceleration relation (MDAR)

\[ g_{\text{bar}}(r) = \frac{GM_{\text{bar}}(<r)}{r^2} \]

McGaugh+16 in PRL
The mass discrepancy-acceleration relation

Two characteristic accelerations:

- $a_0 \sim 10^{-10}$ m/s$^2$: above which there is little need for dark matter, and
- $a_{\text{min}} \sim 10^{-11}$ m/s$^2$ $\sim cH_0$: a "minimum" acceleration probed by galaxies

For reference: Earth’s acceleration around the Sun is $\sim 6 \times 10^{-3}$ m/s$^2$

$cH_0$ is $\sim 7.2 \times 10^{-10}$ m/s$^2$

At the solar circle is $\sim 2 \times 10^{-10}$ m/s$^2$
The mass discrepancy-acceleration relation

Two characteristic accelerations:

- \( a_0 \approx 10^{-10} \text{ m/s}^2 \): above which there is little need for dark matter, and

- \( a_{\text{min}} \approx 10^{-11} \text{ m/s}^2 \approx cH_0 \): a “minimum” acceleration probed by galaxies

For reference: Earth’s acceleration around the Sun is \( \approx 6 \times 10^{-3} \text{ m/s}^2 \)

\( cH_0 \) is \( \approx 7.2 \times 10^{-10} \text{ m/s}^2 \)

(a proxy for surface brightness)
Where do the characteristic accelerations come from?

Every galaxy has a characteristic baryonic acceleration ("$g_{\text{bar}}$") which depends on how its stellar mass and size correlate. This, together with the characteristic acceleration of the halo, which depends on its virial mass and concentration, imply a tight relation between $g_{\text{tot}}$ and $g_{\text{bar}}$ in LCDM.

Navarro+17
LCDM halos have a well-defined maximum central acceleration $a_0 \sim 10^{-10}$ m/s$^2$ is the central acceleration of the most massive halo that may host a disk galaxy ($V_{\text{max}} \sim 200-300$ km/s) $a_{\text{min}} \sim 10^{-11}$ m/s$^2$: is roughly the acceleration of the least massive halo able to host a luminous galaxy ($V_{\text{max}} \sim 20-30$ km/s)
The mass discrepancy-acceleration relation in EAGLE/APOSTLE

Two characteristic accelerations:
- $a_{0} \sim 10^{-10}$ m/s$^2$: above which there is little need for dark matter,
- $a_{\text{min}} \sim 10^{-11}$ m/s$^2$ \( \sim c_{H0} \): a "minimum" acceleration probed by galaxies

For reference: Earth’s acceleration around the Sun is $\sim 6 \times 10^{-3}$ m/s$^2$

\[ g_{\text{bar}}(r) = GM_{\text{bar}}(<r)/r^2 \] (a proxy for surface brightness)
For NFW halos, 

$$a_{\text{max}} \sim f(c) V_{200}^2/r_{200} \sim f(c)^*100*H_0^2*r_{200}^2/a_{\text{max}}/(cH_0) \sim (V_{200}/1000 \text{ km/s})^{(f(c)/30)}(V_{200}/1000 \text{ km/s}) \sim (V_{200}/1000 \text{ km/s})^{a_0 \sim 10^{-10} m/s^2}$$

is the central acceleration of the most massive halo that may host a disk galaxy ($V_{\text{max}} \sim 200-300 \text{ km/s}$).

$a_{\text{min}} \sim 10^{-11} m/s^2$ is roughly the acceleration of the least massive halo able to host a luminous galaxy ($V_{\text{max}} \sim 20-30 \text{ km/s}$).
The small scatter in the relation is partly due to the mass modelling adopted in the SPARC papers, together with the LINEAR sampling of the rotation curves. Note also that various points in a same galaxy are NOT independent from each other so it is unclear what the rms actually means.

Navarro+17