Halometry from Astrometry

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Neal Weiner
New York University
Simons Symposium
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half-baked?

Or delicious chocolate filling?
The importance of gravitational searches

• Dark matter couples to gravity

• Gravitational searches provide strong constraints on what DM is like (interaction strength, non-relativistic)

• Complementary to particle searches: wide net/low specificity

• Capable of testing broad range of DM dynamics
Summary of this talk

• Idea: use weak lensing to look for dark matter (Not new!)

• Idea: use time domain lensing to look for dark matter. Specifically, objects in/around the MW Halo. (Not new!)

• New: High precision astrometric data

• New: Use many sources + high precision + future
Generically: we are going to look for regions of high mass density contrast in/around the milky way

- Compact or diffuse
What would you look for?

• NFW halos - scale dependence? Cutoff?

• Primordial black holes/DM clumps

• High density or ultra dense halos - large adiabatic perturbations, isocurvature perturbations, dissipative DM dynamics

• Dark stars - bosons can condense, fermions can cool and collapse (mirror matter, e.g.)

• “Dark” solar system objects
What lies beneath the light?
What’s changing?

• Gaia
  • An optical, space-based mission that will achieve precision $O(100\mu as)$ over several years
  • 1 billion stars

• Future:
  • Theia - A followup to Gaia that will improve sensitivity by $O(10)$
  • Square Kilometer Array - Radio telescope that will measure $(10^7-10^8)$ quasars to $O(10\mu as)$
Stochastic Noise

\[ \sigma_{\delta \theta}[\mu as] \]
\[ \sigma_{\mu}[\mu as/yr] \]
\[ \sigma^{\text{disp}}_{\mu}[\mu as/yr^2] \]

apparent magnitude
Looking for dark matter in lensing

Video Credit: Frank Summers (STScI)
Weak lensing

\[ \Delta \theta \sim \frac{4GM}{b} \approx 4 \text{ mas} \left( \frac{M}{10^6 M_\odot} \right) \left( \frac{10 \text{kpc}}{b} \right) \]
NFW lens angle template: \(-\hat{\beta}_{\text{il}}(\beta_s/\beta_{\text{il}})G_0[\beta_{\text{il}}/\beta_s]\)
Numerics

\[ \frac{4GM}{b} \rightarrow \frac{4G M_0}{10^{-3} \text{pc}} \sim 40 \text{ mas} \]

\[ \frac{4G (10^8 M_0)}{1 \text{pc}} \sim 4 \text{ mas} \]

\[ \frac{4G (10^8 M_0)}{0.1 \text{pc}} \sim 40 \text{ mas} \]

BH

NFW

\[ \frac{1}{10} \text{ condensed NFW} \]
Time Domain

2 Regimes:

\[ \frac{5b}{6} \leq 1 \]
Weak lensing in the time domain

- "Blips" - Dominik + Sahu 1999, Belokurov + Evans 2002, Erickcek + Li 2011

- Objects move at $\sim 10^{-3}c$, so $\delta b \sim v \tau \sim 10^{-3} \text{ pc}$

- Only relatively light ($\sim M_\odot$ mass) have such close encounters
Weak lensing in the time domain

\[ \frac{\delta \varphi}{b} < 1 \]

\[ \Delta \theta - 46 M \frac{\nu}{b} \]

\[ \Delta \dot{\theta} - 46 M \left( \frac{\nu}{b} \right)^2 \]
Weak lensing in the time domain

\[ \frac{5b}{b} < 1 \]

\[ \Delta \theta \sim \frac{46M}{b} \frac{v}{b} \]

\[ \sim \frac{46M}{(10^{-3} \text{ pc})} \frac{10^{-3}c}{(10^3 \text{ pc})} \sim 10 \text{ mas/yr} \]

\[ \sim \frac{46(10^8M_\odot)}{1 \text{ kpc}} \frac{10^{-3}c}{1 \text{ kpc}} \sim 10^{-3} \text{ mas/yr} \]

\[ \sim \frac{46(10^8M_\odot)}{0.1 \text{ kpc}} \frac{10^{-3}c}{(1 \text{ kpc})} \sim 0.1 \text{ mas/yr} \]
Lensing in the instantaneous approximation

\[
\Delta \dot{\theta}_{il} \sim \frac{4G NM_s v_{il}}{b_{il}^2} \min \left[ 1, \left( \frac{b_{il}}{r_s} \right)^{3-n} \right]; \quad \Delta \ddot{\theta}_{il} \sim \frac{4G NM_s v_{il}^2}{b_{il}^3} \min \left[ 1, \left( \frac{b_{il}}{r_s} \right)^{3-n} \right]
\]

- Velocity - instrumental & intrinsic noise
- Acceleration - dominated by instrumental noise
These effects are tiny
New opportunities with many sources

- Two new possibilities:
  - **Rare objects**: in the presence of many objects, close encounters become possible
    - Blips - A lens passes near a source with $\delta b / b \sim 1$
    - Outlier - $\delta b / b < 1$ but lens is sufficiently close that acceleration or velocity is above expectations
  - **Multi-object observables**: effects on individual sources are below noise, but in aggregate can be visible
    - Multi-blips - A lens passes near many sources with $\delta b / b \sim 1$
    - Templates - small effects are matched to an expected pattern arising from a diffuse halo
    - Correlations - no large scale patterns, but “nearby” sources have motions or accelerations correlated with each other
Examples

- **Rare objects**: Blips - A lens passes near a source with $\delta b / b \sim 1$
  
- Challenges: not in current data set
  
- Not repeatable
Examples

- **Rare objects**: Outlier - $\delta b/b < 1$ but lens is sufficiently close that acceleration or velocity is above expectations

- Challenges: uncertainties in expectations
Examples

- **Multi-object observables**: Multi-blips
  - A lens passes near many sources with \( \frac{\delta b}{b} \sim 1 \) (note this is also a rare object)

- This is also a form of a template in the time domain

- Not in current data set
Looking for diffuse objects

BUT CAN YOU LOOK FOR OBJECTS THIS BIG

CHILL MAN, I GOT THIS
Examples - Template

- **Multi-object observables**: Templates - small effects are matched to an expected pattern arising from a diffuse halo.

- Even with large numbers of objects, sensitive to noise.

![Diagram showing NFW lens velocity template: $G_1[\beta_{ii}/\beta_s]$](image)

Template
Examples

- **Multi-object observables**: Correlations - no large scale patterns, but “nearby” sources have motions or accelerations correlated with each other.

- Challenges - things have correlations!
Luminous Sources

- Disk stars
  - Lots of them, bright (close)
  - High angular velocity (close), less DM in LOS (close)
- LMC/SMC
  - Not as many but still plenty ($10^7$), poor angular precision (far)
  - Concentrated in small angular region (so high density)
  - Low angular velocity (far), lots of DM in LOS (far)
- Quasars
  - Not as many ($10^6$), Faint (tough to resolve optically)
  - Stable, uncorrelated
Blips and Multiblips

\[ B [x_l(t)] = \sum_{i \in \Box} \frac{1}{\sigma_{\delta \theta, i}^2} \sum_n \delta \theta_i(t_n) \cdot \Delta \theta_{il}[x_l(t_n)]. \]

Requires many sources+objects for close approaches
Outliers

\[ \mathcal{O}_\mu \equiv \max_i \frac{\dot{\theta}_i^2}{2\sigma_{\mu,i}^2} \quad \text{and} \quad \mathcal{O}_\alpha \equiv \max_i \frac{\dot{\theta}_i^2}{2\sigma_{\alpha,i}^2} \]

\[ \Delta \dot{\theta}_{il} \sim \frac{4G_N M_s v_{il}}{b_{il}^2} \min \left[ 1, \left( \frac{b_{il}}{r_s} \right)^{3-n} \right] \]

Most sensitive just above the “blip” regime
Compact Object Sensitivity

\[ \log_{10}[\rho/\rho_c] \]

\[ \log_{10}[M_*/M_\odot] \]

- mono-blips
- multi-blips
- outlier acceleration
- outlier velocity

\[ B^{\text{multi}} \]
\[ B^{\text{mono}} \]
Looking for diffuse objects

- Velocity effects are *small*
- Individual motions are sub-noise
- Need a lot of objects
Templates

\[ T_\mu [\theta_t, \beta_t, \hat{v}_t] = \sum_i \frac{\dot{\theta}_i}{\sigma_{\mu,i}^2} \cdot \mu_t \left[ \frac{\beta_{it}}{\beta_t}, \hat{\beta}_{it}, \hat{v}_t \right]; \quad \beta_{it} \equiv \theta_t - \theta_i. \]

Requires high density of sources with low noise

LMC \rightarrow \text{quasars}
Correlations

\[ C_\mu[\beta_-, \beta_+, \delta] = \frac{1}{2} \sum_{i \neq j} \frac{\dot{\theta}_i \cdot \dot{\theta}_j}{\sigma_{\mu,i}^2 \sigma_{\mu,j}^2} \frac{B[\beta_{ij}; \beta_-, \beta_+] \beta_{ij}^\delta}{\beta_{ij}^6} \]

\[ C_\alpha[\beta_-, \beta_+, \delta] = \frac{1}{2} \sum_{i \neq j} \frac{\dddot{\theta}_i \cdot \dddot{\theta}_j}{\sigma_{\alpha,i}^2 \sigma_{\alpha,j}^2} \frac{B[\beta_{ij}; \beta_-, \beta_+] \beta_{ij}^\delta}{\beta_{ij}^6} \]

Requires high density of sources with low noise and low intrinsic correlations

\( \alpha - \text{Disk stars} \quad M - \text{quasars} \)
NFW Subhalos

correlated acceleration  template velocity  correlated velocity

\[
\log_{10}[r_s/\text{pc}] = \alpha
\]

\[
\log_{10}[M_s/M_\odot] = \mu
\]
NFW Subhalos

correlated acceleration  template velocity  correlated velocity

$r_s < \sigma_{v_1} \tau$

$z_{\text{coll}} = 300$

$z_{\text{coll}} = 100$

$z_{\text{coll}} = 30$

Standard NFW

$\log_{10}[\rho_s/M_\odot\text{pc}^{-3}]$

$\log_{10}[M_s/M_\odot]$
LMC 0.2° radius

Sources

Excess Noise
LMC 0.2° radius

Sources

Excess noise
# Systematic Noise

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The history of dark matter

Urbain Le Verrier
Solar System Objects

- Objects beyond the Kuiper belt can be difficult to see
- Reflected light $\sim D^{-4}$, IR emission $\sim D^{-2}$
- Hard to do long exposure
- Deeply ignorant of what is beyond the KB (deeply!)
EVIDENCE FOR A DISTANT GIANT PLANET IN THE SOLAR SYSTEM

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ABSTRACT

Recent analyses have shown that distant orbits within the scattered disk population of the Kuiper Belt exhibit an unexpected clustering in their respective arguments of perihelion. While several hypotheses have been put forward to explain this alignment, to date, a theoretical model that can successfully account for the observations remains elusive. In this work we show that the orbits of distant Kuiper Belt objects (KBOs) cluster not only in argument of perihelion, but also in physical space. We demonstrate that the perihelion positions and orbital planes of the objects are tightly confined and that such a clustering has only a probability of 0.007% to be due to chance, thus requiring a dynamical origin. We find that the observed orbital alignment can be maintained by a distant eccentric planet with mass $\gtrsim 10 m_{\oplus}$ whose orbit lies in approximately the same plane as those of the distant KBOs, but whose perihelion is $180^\circ$ away from the perihelia of the minor bodies. In addition to accounting for the observed orbital alignment, the existence of such a planet naturally explains the presence of high-perihelion Sedna-like objects, as well as the known collection of high semimajor axis objects with inclinations between 60° and 150° whose origin was previously unclear. Continued analysis of both distant and highly inclined outer solar system objects provides the opportunity for testing our hypothesis as well as further constraining the orbital elements and mass of the distant planet.

Key words: Kuiper Belt: general – planets and satellites: dynamical evolution and stability

Looking for Planet Nine, Astronomers Gaze into the Abyss

Two years on, the search for our solar system’s missing world is as frenzied as ever—and the putative planet is running out of places to hide

By Lee Billings on March 22, 2018
Remote Earth Mass Objects from Variable Astrometry

\[ \Delta \theta_{il} = \frac{4GM_l}{b_{il}^*} \approx 0.024 \, \mu\text{as} \left[ \frac{M_l}{M_\oplus} \right] \left[ \frac{\text{AU}}{b_{il}} \right]. \]

\[ N_0^B \approx \frac{4\Sigma_0 \text{AU}^2}{R_l^2} \approx 400 \left[ \frac{\Sigma_0}{10^8} \right] \left[ \frac{1000 \, \text{AU}}{R_l} \right]^2. \]

\[ \text{SNR}_{\text{multi}} \approx \frac{4GM_l}{R_l} \frac{\sqrt{f_{\text{rep}}}}{\sigma_{\delta \theta, \text{eff}}} \sqrt{2\pi \Sigma_0 \ln \left[ \frac{4\Sigma_0 \text{AU}^2}{R_l^2} \right]} \].

Sensitivity goes like $1/R$
Planet Nine

Multi-blip searches for outer Solar System planets

$R [\text{AU}]$

$M [M_\oplus]$
The near future

- Gaia data is out (DR2)
- Is a real data set
- Parallaxes not converged to final accuracy
- Many sources of noise to be understood
- Near term: outlier velocities, template velocities
The far future

- 10-20 μas measurements of quasars possible
- WFIRST?
- Proper motion measurements is a long game
Conclusions

- Precision + time domain + astrometry = new dark opportunities
  - We are just scratching the surface
- Simple observables - mono blips, outliers - can provide significant sensitivity to compact objects
- Multi-object observables (multi-blips, templates, correlations), allow sensitivity to effects that are individually unobservable
- We can get sensitivity to a wide range of dark objects, including NFW halos, high density halos
- Planet 9!
Backup
Universality Classes

\[ \rho(r) = \frac{2^{3-\gamma} \rho_s}{\left( \frac{r}{r_s} \right)^\gamma \left( 1 + \frac{r}{r_s} \right)^{3-\gamma}} \]

\[ \Delta \dot{\theta}_{il} \sim \frac{4G_N M_s v_{il}}{b_{il}^2} \min \left[ 1, \left( \frac{b_{il}}{r_s} \right)^{\min[2,3-\gamma]} \right] \]

\[ \Delta \ddot{\theta}_{il} \sim \frac{4G_N M_s v_{il}^2}{b_{il}^3} \min \left[ 1, \left( \frac{b_{il}}{r_s} \right)^{\min[2,3-\gamma]} \right] \]

“soft lens”: \( \gamma < 2 \) \hspace{1cm} “hard lens”: \( \gamma \geq 2 \)
Systematics: outliers

accelerations: disk vs Magellanic Clouds:

\[ \dot{\theta}_{ic} = \frac{G^{1/3}}{D_i} \frac{m_c}{m_i^{2/3}} \left( \frac{2\pi}{T} \right)^{4/3} \]

\[ \approx 0.18 \, \mu\text{as} \, y^{-2} \left[ \frac{10 \, \text{kpc}}{D_i} \right] \left[ \frac{m_c}{10^{-3} M_\odot} \right] \left[ \frac{M_\odot}{m_i} \right]^{2/3} \left[ \frac{10 \, y}{T} \right]^{4/3} \]

velocities: nearby proper motions + line-of-sight velocity
Systematics: blips

**mono-blips:** disk vs Magellanic Clouds
mass determination

**multi-blips:** follow-up optical studies
Systematics: correlations

accelerations:
\[ \alpha_d[D_i, R, z] \approx \frac{G_N \Sigma_{d,0}}{2D_i} e^{-\frac{R}{R_d}} \left[ 1 - e^{-\frac{|z|}{z_d}} \right] \]
\[ \approx 2 \times 10^{-6} \muas y^{-2} \left[ \frac{10 \text{ kpc}}{D_i} \right] \left[ \frac{e^{-\frac{R}{R_d}}}{e^{-\frac{R}{R_d}}} \right] \left[ 1 - e^{-\frac{|z|}{z_d}} \right] \]

velocities: scale separation + template checks + extra-galactic sources
Look-Elsewhere Effect

\[ \text{SNR}^{\text{global}} \approx \frac{\text{SNR}^{\text{local}}}{\sqrt{1 + \ln N_{\text{trial}}}} \]

\[ \sqrt{1 + \ln N_{\text{trial}}} \]

\[ \begin{align*}
\lesssim 4.7 & \quad (O_{\mu}, O_{\alpha}) \\
\approx 5.4 & \quad (B^\text{mono}) \\
\approx 4.1 & \quad (B^\text{multi}) \\
\lesssim 3.5 & \quad (T_{\mu}) \\
= 1 & \quad (C_{\mu}, C_{\alpha})
\end{align*} \]
SNR: mono-blip

\[
\left\langle \min_{i,l} b_{il}^* \right\rangle = \frac{M_l}{\sigma_{v_l} \tau \rho_l D_i \Sigma_0 \Delta \Omega} \lesssim \sigma_{v_l} \tau
\]

\[
\text{SNR}^{\text{mono}}_B \simeq \frac{4G_N M_l}{\sigma_{\delta \theta, \text{eff}}} \sqrt{\pi f_{\text{rep}}} \frac{1}{\sigma_{v_l}} \frac{1}{\left\langle \min_{i,l} b_{il}^* \right\rangle}
\]
SNR: velocity template

\[
\text{SNR}_{\tau_{\mu}} [D_l, v_{il}] = C_1 \left(1 - \frac{D_l}{D_i}\right) \frac{4 G_N M_s v_{il}}{r_s D_l} \sqrt{\frac{\Sigma_0}{\sigma_{\mu,\text{eff}}}}
\]

\[
\left\langle \min_l D_l \right\rangle = \left(\frac{3}{n_l \Delta \Omega}\right)^{1/3} \approx 18 \text{ kpc} \left[\frac{M_s}{10^7 M_\odot} \frac{1}{\Omega_{\text{sub}}} \frac{0.01}{\Delta \Omega}\right]^{1/3}
\]

\[
\left\langle \max_l \text{SNR}_{\tau_{\mu}} \right\rangle \approx \frac{\pi^{1/2} C_1}{2^{1/2} 3^{1/3}} \frac{4 G_N M_s \sigma_{v_l}}{r_s} (n_l \Delta \Omega)^{1/3} \sqrt{\frac{\Sigma_0}{\sigma_{\mu,\text{eff}}}} \approx 0.4 \Omega_{\text{sub}}^{1/3} \left[\frac{M_s}{10^7 M_\odot}\right]^{2/3} \left[\frac{10 \text{ pc}}{r_s}\right] \left[\frac{N_0}{10^7}\right]^{1/2} \left[\frac{0.01}{\Delta \Omega}\right]^{1/6} \left[\frac{200 \mu\text{as}}{\sigma_{\mu,\text{eff}}}\right]
\]
SNR: correlations

\[ \text{SNR}_C = \sqrt{\frac{\pi}{2}} \sqrt{\frac{2\Delta \Omega}{\Sigma_0}} \left( n_l r_s^3 \right) \left( \frac{4G_N M_s v_{il}}{r_s^2 \sigma_{\mu, \text{eff}}} \right)^2 I_1 \left[ \frac{\beta_-}{r_s/D_{\text{max}}}, \frac{\beta_+}{r_s/D_{\text{max}}}, \delta, \epsilon_s \right] \]

\[ I_1 [z_-, z_+, \delta, \epsilon_s] \equiv \sqrt{\frac{2 - 2\delta}{z_+^{2-2\delta} - z_-^{2-2\delta}}} \int_{z_-}^{z_+} dz z^{1-\delta} \int_0^1 dy \int_{x>zy+\epsilon_s} d^2x \left| G_1[x, \hat{x}, \hat{v}] \right|^2 \]

\[ \text{SNR}_{C \alpha} = \sqrt{\frac{\pi}{2}} \sqrt{\frac{2\Delta \Omega}{\Sigma_0}} \left( n_l r_s^3 \right) \left( \frac{4G_N M_s v_{il}^2}{r_s^3 \sigma_{\alpha, \text{eff}}} \right)^2 I_2 \left[ \frac{\beta_-}{r_s/D_i}, \frac{\beta_+}{r_s/D_i}, \delta, \epsilon_s \right] \]

\[ I_2 [z_-, z_+, \delta, \epsilon_s] \equiv \sqrt{\frac{2 - 2\delta}{z_+^{2-2\delta} - z_-^{2-2\delta}}} \int_{z_-}^{z_+} dz z^{1-\delta} \int_0^1 dy \left( 1 - y \right)^2 \int_{x>zy+\epsilon_s} d^2x \left| G_2[x, \hat{x}, \hat{v}] \right|^2 \]
SNR: correlations

- \[ h_i(z_-, z_+, \delta, \tau) \]
- \[ \log_{10}[r]/[pc] \]
- \[ \delta = 0.70, r_s = 10^2 [pc], \tau = 5y \]
- \[ \delta = 0.85, r_s = 10^{-1} [pc], \tau = 5y \]
NFW templates

azimuthal average of square template vector

NFW lens angle template: $-\beta_{il}(\beta_s/\beta_l)G_0[\beta_l/\beta_s]$

NFW lens velocity template: $G_1[\beta_{ll}/\beta_s]$

NFW lens acceleration template: $G_2[\beta_{ll}/\beta_s]$