IR structure in cosmology and amplitudes

Enrico Pajer

What happens at soft momenta?

\[ \lim_{p_1 \to 0} A_n(p_1, \ldots, p_n) = ? \]
\[ \lim_{|p_1| \to 0} B_n(p_1, \ldots, p_n) = ? \]

where \( B_n \) are the equal-time correlators

\[ \langle O_1(\vec{p}_1) \ldots O_n(\vec{p}_n) \rangle = (2\pi)^3 \delta^3(\sum_{a=1}^{n} p_a) B_n \].

Soft limits \( \leftrightarrow \) non-linearly realized symmetries

\[ \mathcal{O} \to \mathcal{O} + \text{const} + \ldots \],

as in gauge theories, gravity and spontaneous symmetry breaking.

\[ \text{IR structure : amplitudes = non-linear symmetries : Lagrangians} \]

Soft amplitudes

Soft scalars

Adler’s zero fo shift-symmetric scalars (e.g. pions)

\[ \lim_{p \to 0} A_n = 0 \]

Scalar EFTs are not fixed by factorization. Need to add soft limits

\[ \lim_{p \to 0} A = \mathcal{O}(p^2), \]

Exceptional EFT’s: sigma model’s, DBI and special Galileons [1].

Soft vectors

Weinberg’s soft-photon theorem

\[ \lim_{q \to 0} A_{\mu}^n \to A_{\mu}^{n-1} \sum_a \frac{e_a f_a}{-p_a \cdot q - i\epsilon}, \]

gauge invariance demands charge conservation

\[ 0 = q_{\mu} A_{\mu}^n = \sum_a e_a \]

Strominger IR triangles [2]: soft theorems \( \leftrightarrow \) memory \( \leftrightarrow \) asymptotic symmetries
Soft gravitons

Weinberg’s soft-photon theorem

\[
\lim_{q \to 0} A^{\mu\nu}_n \to A_{n-1} - \sum_a \frac{f_a p_a^\mu p_a^\nu}{-p_a \cdot q - i \epsilon},
\]  

(10)

Diff invariance demands universal coupling

\[
0 = q_{\mu} A_n^{\mu} = \sum_a f_a p_a^\mu \Rightarrow f_a = \sqrt{8 \pi G N}
\]  

(11)

Higher spin particles \( s \geq 3 \) are forbidden to interact by a similar argument.

- IR divergences and IR safe observables
- Subleading and multiple soft limits, group structure, ...
- Phenomenology: Dark Energy, massive/modified gravity

Cosmological correlators

Spontaneously broken boosts and time-translations (flat FLRW)

\[
ISO(3, 1) \to ISO(3),
\]

(12)

Poincaré in 3+1 \( \to \) Euclidean in 3 .

(13)

Adiabatic modes

Weinberg’s prescription (make drawing) [3]:

1. Fix small diffs

\[
x^\mu \to x^\mu + \epsilon^\mu \quad \text{with} \quad \lim_{x \to \infty} \epsilon^\mu = 0
\]

(14)

2. Find residual large diff’s \( \lim_{x \to \infty} \epsilon^\mu = \infty \) supported at \( q = 0 \). They solve EE’s

3. Some large diffs extend to \( q \neq 0 \): adiabatic modes are physical solutions that are locally indistinguishable from a change of coordinates

There are scalar, vector, tensor adiabatic modes and mixes thereof [11, 6] Consequences:

- Classical: Soft, model-independent solutions to EE’s, typically \( q \ll H \)
- Quantum: non-linear symmetry

\[
\Delta \zeta \sim \frac{1}{3} \partial_t \epsilon^t + \epsilon^i \partial_i \zeta
\]

(15)

and associated soft theorems

Soft theorems

\[
\lim_{q \to 0} \frac{\langle O(q)O(k_1)\ldots O(k_n) \rangle'}{\langle O(q)O(q) \rangle'} = \sum_{a=1}^{n} L_a \langle O(k_1)\ldots O(k_n) \rangle',
\]

(16)

Many ways to derive them:

- Background wave method (semi-classical)
- Ward-Takahashi identities for symmetry induced by adiabatic modes
- OPE coefficients fixed by symmetry
- Wave function \( \Psi[\zeta] \sim \Psi[\zeta + \Delta \zeta] \)
Maldacena’s soft theorem

Maldacena’s single-field consistency relation [4, 5, 10]

\[ \lim_{q \to 0} \langle \zeta(q) \zeta(k) \zeta(k') \rangle' = (1 - n_s) P(q) P(k) + O(q^2) \]  

is WT identity of \( \epsilon^{i} = \lambda x^{i} \) with symmetry [7, 8, 9]

\[ \zeta(x) \to \zeta(x) + \lambda [1 + x^{i} \partial_{i} \zeta(x)] \]  

\[ \zeta(k) \to \zeta(k) + \lambda (2\pi)^{3} \delta^{3}_{D}(k) - (3 + k^{i} \partial_{k^{i}} \zeta(k)) \]  

- Generalized to all n-point and to all orders in \( \epsilon^{i} \sim O(x^{n}) \)
- Analogous soft tensor theorems are more robust because of Higuchi bound

How Gaussian can our universe be?

What is the lowest level of non-Gaussianity? In multifield \( f_{NL}^{loc} \sim O(1) \). Non-canonical single field \( f_{NL}^{eq} \sim c_{s}^{-2} \). The most Gaussian models are canonical single-field slow-roll:

\[ B_{3}^{single-field} \sim (1 - n_s) B_{3}^{local} + \epsilon B_{3}^{equil} . \]  

By construction the soft limit is not locally observable (locally a change of coords). In fact, the physical distance and momentum are

\[ g_{ij} = a^{2} e^{2 \zeta} dx^{i} dx^{j} \]  

\[ \Delta x^{2}_{phys} = \Delta x^{2} a^{2} e^{2 \zeta} \Rightarrow k_{phys} = \frac{k}{a(1 + \zeta)} \sim \frac{k}{a} (1 - \zeta) \]  

Then the bispectrum in physical momenta vanishes

\[ \lim_{q \to 0} \langle \zeta(q) \zeta(k_{phys}) \zeta(k'_{phys}) \rangle' \simeq (1 - n_s) P(q) P(k) + \langle \zeta(q) \zeta(k \cdot \partial_{k} \zeta(k) \zeta(k') + \zeta(k) k' \cdot \partial_{k'} \zeta(k') \rangle \]  

\[ \simeq [(1 - n_s) + (n_s - 1)] P(q) P(k) + O(q^2) = 0 . \]  

But a term

\[ B_{3}^{minimum} \sim (n_s - 1) \frac{q^{2}}{k^{2}} P(q) P(k) \frac{2(1 + 3 \cos \theta)^{2}}{10} \]  

survives and establishes the lower bound on inflationary non-Gaussianity [15] (barring fine tuning). Three ways to see this (see Sec. 5 of [15])

1. Direct computation in Conformal Fermi Coordinates

2. In spatially-flat gauge

\[ \langle \phi^{3} \rangle \propto \epsilon , \]  

because \( \phi \) is minimally coupled and can only see Riemann

\[ R_{\mu\nu\sigma\tau} = H^{2} (g_{\mu\nu} g_{\sigma\tau} - g_{\mu\sigma} g_{\nu\tau}) + O(\epsilon^{2} \zeta) \]  

But changing variables to \( \zeta \) gives

\[ \phi \sim \zeta + \eta \zeta^{2} + \ldots \]  

and so

\[ \langle \zeta^{3} \rangle \propto \eta + O(\epsilon) . \]  

3. Work directly in \( \zeta \) gauge. Then action for \( \zeta(k) \) in background of \( \zeta(q) \) is

\[ \mathcal{L} \sim \epsilon \partial_{k}^{2} \zeta + \epsilon (1 + \eta) \zeta_{q}^{2} \zeta_{k}^{2} \]  

3
Violation of Maldacena’s consistency relation

Maldacena’s soft theorem is not respected (i.e. assumptions are violated) in the following interesting cases:

• Spatial curvature: corrections

\[ B(q_l \ll k_s) \sim P_l P_s \left[ (1 - n_s) + \frac{(1 - c_s^2)(1 - n_s) K}{q_l^2} + \frac{1 - c_s^2}{k_s^2} K \right] \]  

(30)

Current bounds

\[ \frac{K}{k_s^2} < \frac{K}{q_l^2} < \Omega_K < 10^{-3} \]  

(31)

• Non-attractor cosmologies: \( \zeta \) is not constant as \( q \to 0 \). Examples: multifield inflation and Ultra-Slow-Roll inflation [12].

• Non-standard symmetry breaking pattern. For example solid inflation avoids the theorem because of large anisotropic stresses, which ensure that anisotropy is not erased (non-attractor).

Generalized adiabatic modes

Generalized Ad. Modes are solutions that are locally indistinguishable from a change of coordinates and an internal symmetry.

• New soft theorem in shift-symmetric cosmologies, including Ultra-Slow-Roll inflation [13]

• New soft theorems in solid inflation [14]

• Extend to all additional symmetries besides diffs

Open questions

• Soft limits from the boundary?

• Soft limit recursion relations for cosmo correlators?

• Coleman-Mandula for cosmology? Classify all soft correlators

References


